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Research Article

Applications of Mohand Transform

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Abstract

Investigating solutions of differential equations has been an important issue for scientists. Researchers around the world have talked about different methods to solve differential equations. The type and order of the differential equation enable us to decide the method that we can choose to find the solution of the equation. One of these methods is the integral transform, which is the conversion of a real or complex valued function into another function by some algebraic operations. Integral transforms are used to solve many problems in mathematics and engineering, such as differential equations and integral equations. Therefore, new types of integral transforms have been defined, and existing integral transforms have been improved. One of the solution methods of many physical problems as well as initial and boundary value problems are integral transforms. Integral transforms were introduced in the first half of the 19th century. The first historically known integral transforms are Laplace and Fourier transforms. Over the time, other transforms that are used in many fields have emerged. The aim of this article is to describe the Mohand transform and to make applications of linear ordinary differential equations with constant coefficients without any major mathematical calculations This integral transform method is an alternative method to existing transforms such as Laplace transform and Kushare transform. When recent studies in the literature are examined, it can be said that Mohand transform is preferred because it is easy to apply.

Keywords: Differential equation, Integral transform, Laplace and Fourier transform.

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1. Introduction

Integral transforms [1-12] such as Laplace, Mohand, Aboodh, Anuj, Kamal, Kushare, Mahgoub, Ara, HY, Sadik etc. have assumed an important role to solve science, engineering, and real life problems. Researchers have been proposed these transforms to solve the problems including differential equations, telegraph differential equations, integral equations, integro-differential equations.

Attaweel and Almassry used Mohand transform, which they considered as a modified version of Laplace and Sumudu transforms to solve ordinary differential equations with variable coefficients [12]. Dehinsulu O.A [13] suggested the Mahgoub transform method to solve linear convection-diffusion problems with constant coefficients. When Dehinsulu compared the results obtained by this method to the exact solutions, he found an excellent agreement. Furthermore, in the year (2022), Abbas E.S. et al. [14] described Aboodh transform to solve some telegraph equations with specific initial conditions. Aggarwal S. et al. [15] used recently developed transform method called Rishi transform to acquire the analytical solution of linear volterra integral equation. Also, Kuffi E. et al. [16] introduced a new integral transform method named Emad-Falih transform to find the solutions of first-order and second-order ordinary differential equations. Gupta R. [17] proposed a novel transform method called Rohit transform to analyze boundary value problems. However, Chaudhary R. and Aggarwal S. [18] presented a comparative work of Laplace and Mohand transforms. Their results in application section demonstrated that both transforms are closely connected. In the year 2023, Mushtt and Kuffi [19] compared the Sadik transform and the complex Sadik transform to solve systems of ordinary differential equations. Integral transforms will continue to be used in many scientific researches because researchers argue that integral transforms give accurate solutions to many complex problems and are applicable in many different fields such as astronomy and mechanics.

2. Mohand Transforms Basic Definitions and Properties

2.1. Mohand transform definition [18, 20, 21]

Mohand transform is described for a function of exponential order in the A set as:

$$A = \left\{ f(t) : \exists\, M\,, k_1, k_2 > 0\,, |f(t)| < Me^{\frac{|t|}{k_j}}\,,\ if\ t \in (-1)^j \times [0, \infty) \right\}$$

where $f(t)$ is a dedicated function in the set A, M is a finite number, and k_1, k_2 can be finite or infinite.

Mohand transform denoted by operator M is defined as:

$$\mathrm{M}\{\mathrm{f(t)}\} = \mathrm{K(v)} = \mathrm{v}^2 \int_0^\infty \mathrm{f(t)e^{-vt}\,dt}, \quad \mathrm{t} \geq 0,\ \mathrm{k_1} \leq \mathrm{v} \leq \mathrm{k_2}.$$

2.2. Linearity feature of Mohand transform [18, 20]

Let $M\{f_1(t)\} = K_1(v)$ and $M\{f_2(t)\} = K_2(v)$. Then Mohand transform of $M\{af_1(t) + bf_2(t)\}$ is given as:

$$M\{af_1(t) + bf_2(t)\} = aK_1(v) + bK_2(v)$$

where a, b are arbitrary constants.

2.3. Scalar variation feature of Mohand transform [12, 18]

Let $M\{f(t)\} = K(v)$. Mohand transform of a function $f(at)$ is defined as:

$$M\{f(at)\} = aK\left(\frac{v}{a}\right).$$

2.4. Scrolling feature of Mohand transform [12, 18]

Let $M\{f(t)\} = K(v)$. Mohand transform of a function $e^{at}f(t)$ is defined as:

$$M\{e^{at}f(t)\} = \frac{v^2}{(v-a)^2}K(v-a).$$

2.5. Convolution theorem for Mohand transform [18, 21]

Let $M\{f_1(t)\} = K_1(v)$ and $M\{f_2(t)\} = K_2(v)$. Then Mohand transform of their convolution $f_1(t) * f_2(t)$ is given as:

$$M\{f_1(t) * f_2(t)\} = \frac{1}{v^2}M\{f_1(t)\}M\{f_2(t)\},$$

$$M\left\{f_1(t) * f_2(t)\right\} = \frac{1}{v^2}K_1(v)K_2(v),$$

where $f_1(t) * f_2(t)$ is described as:

$$f_1(t) * f_2(t) = \int_0^t f_1(t-x)\, f_2(x)dx = \int_0^t f_1(x)\, f_2(t-x)dx.$$

2.6. Mohand transform of the derivatives [18, 22]

Let $M\{f(t)\} = K(v)$. Mohand transforms of the derivatives of a function of $f(t)$ are given as:

- $M\{f'(t)\} = vK(v) - v^2f(0)$
- $M\{f''(t)\} = v^2K(v) - v^3f(0) - v^2f'(0)$
- $M\left\{f^{(n)}(t)\right\} = v^nK(v) - v^{n+1}f(0) - v^nf'(0) - \cdots - v^2f^{(n-1)}(0)$

2.7. Mohand transform of the integral [18, 22]

Let $M\{f(t)\} = K(v)$. Mohand transform of the integral of a function $f(t)$ is given as:

$$M\left\{\int_0^t f(t)\, dt\right\} = \frac{1}{v}K(v).$$

3. Mohand Transform and Inverse Mohand Transform of Some Elementary Functions [12, 20]

- $M\{1\} = v = K(v)$

  Inversion formula: $M^{-1}\{v\} = 1 = f(t)$
- $M\{t^n\} = \frac{n!}{v^{n-1}} = K(v) \quad , \quad n \in N$

  Inversion formula: $M^{-1}\left\{\frac{n!}{v^{n-1}}\right\} = t^n = f(t)$
- $M\{e^{at}\} = \frac{v^2}{v-a} = K(v)$

  Inversion formula: $M^{-1}\left\{\frac{v^2}{v-a}\right\} = e^{at} = f(t)$
- $M\{sinat\} = \frac{av^2}{v^2+a^2} = K(v)$

  Inversion formula: $M^{-1}\left\{\frac{av^2}{v^2+a^2}\right\} = sinat = f(t)$

- $M\{cosat\} = \frac{v^3}{v^2+a^2} = K(v)$

Inversion formula: $M^{-1}\left\{\frac{v^3}{v^2+a^2}\right\} = cosat = f(t)$

- $M\{sinhat\} = \frac{av^2}{v^2-a^2} = K(v)$

Inversion formula: $M^{-1}\left\{\frac{av^2}{v^2-a^2}\right\} = sinhat = f(t)$

- $M\{coshat\} = \frac{v^3}{v^2-a^2} = K(v)$

Inversion formula: $M^{-1}\left\{\frac{v^3}{v^2-a^2}\right\} = coshat = f(t)$

4. Applications of Ordinary Differential Equations of First and Second Order by Mohand Transform

Assume that the first-order ordinary differential equation with the initial condition $y(0) = a$ is given as

$$\boldsymbol{\frac{dy}{dt} + ky = g(t)} \quad , \quad \boldsymbol{t > 0} \tag{1}$$

where Mohand transform of $g(t)$ as a function of "t" is denoted by $G(v)$ and a, k are constants.
Applying Mohand transform on both side in equation (1), we have

$$M\left\{\frac{dy}{dt}\right\} + kM(y) = M\{g(t)\}$$

$$vM(y) - v^2y(0) + kM(y) = G(v)$$

$$vM(y) + kM(y) = G(v) + av^2$$

$$M(y) = \frac{G(v)}{(v+k)} + \frac{av^2}{(v+k)}$$

Then we find the solution by applying the inverse Mohand transform in the step above.
Assume that the second-order ordinary differential equation with the initial conditions $(0) = a$, $y'(0) = b$ is given as

$$\boldsymbol{\frac{d^2y}{dt^2} + k\frac{dy}{dt} + ly = g(t)} \quad , \quad \boldsymbol{t > 0} \tag{2}$$

where Mohand transform of $g(t)$ as a function of "t" is
denoted by $G(v)$ and a, b, k, l are constants.
Applying Mohand transform on both side in equation (2), we have

$$M\left\{\frac{d^2y}{dt^2}\right\} + kM\left\{\frac{dy}{dt}\right\} + lM(y) = M\{g(t)\}$$

$$\{v^2M(y) - v^3y(0) - v^2y'(0)\} + k\{vM(y) - v^2y(0)\} + lM(y) = G(v)$$

$$M(y)(v^2 + kv + l) = G(v) + av^3 + v^2(b + ak)$$

$$M(y) = \frac{G(v)}{(v^2+kv+l)} + \frac{av^3}{(v^2+kv+l)} + \frac{v^2(b+ak)}{(v^2+kv+l)}$$

Then we find the solution by applying the inverse Mohand transform in the step above.
Example. Assume that the first-order differential equation

$$\boldsymbol{y' + 27y = \cos 9t}, \quad \boldsymbol{y(0) = 0} \tag{3}$$

Applying Mohand transform on both side in equation (3), we get

$$M\{y'\} + 27M(y) = M\{\cos 9t\}$$

$$vM(y) - v^2y(0) + 27M(y) = \frac{v^3}{v^2 + 81}$$

$$M(y) = \frac{v^3}{(v^2 + 81)(v + 27)}$$

$$M(y) = \frac{1}{30}\frac{v^3}{(v^2 + 81)} + \frac{1}{10}\frac{v^2}{(v^2 + 81)} - \frac{1}{30}\frac{v^2}{(v + 27)}$$

The inverse Mohand transform of the equation above gives us the solution:

$$y(t) = \frac{1}{30}\ cos9t + \frac{1}{90}\ sin9t - \frac{1}{30}\ e^{-27t}$$

Example. Assume that the second-order differential equation

$$\boldsymbol{y'' + y = 0 \quad , \quad y(0) = y'(0) = 1} \tag{4}$$

Applying Mohand transform on both side in equation (4), we have

$$M\{y''\} + M(y) = 0$$

$$v^2M(y) - v^3y(0) - v^2y'(0) + M(y) = 0$$

$$(v^2 + 1)M(y) = v^3 + v^2$$

$$M(y) = \frac{v^3}{v^2 + 1} + \frac{v^2}{v^2 + 1}$$

The inverse Mohand transform of the equation above is simply obtained as:

$$y(t) = cost + sint$$

Example. Consider the following equation:

$$\boldsymbol{y' + 13y = e^{11t} \quad , \quad y(0) = 1} \tag{5}$$

Applying Mohand transform on both side in equation (5), we have

$$M\{y'\} + 13M(y) = M\{e^{11t}\}$$

$$vM(y) - v^2y(0) + 13M(y) = \frac{v^2}{v - 11}$$

$$(v + 13)M(y) = \frac{v^2}{v - 11} + v^2$$

$$M(y) = \frac{v^2(v - 10)}{(v - 11)(v + 13)}$$

$$M(y) = \frac{1}{24}\frac{v^2}{(v - 11)} + \frac{23}{24}\frac{v^2}{(v + 13)}$$

The inverse Mohand transform of the equation above gives us the solution:

$$y(t) = \frac{1}{24}e^{11t} + \frac{23}{24}\ e^{-13t}$$

Example. Assume that the following equation

$$\boldsymbol{y'' - 3y' + 2y = 0 \quad , \quad y(0) = 1\ , \quad y'(0) = 4} \tag{6}$$

Applying Mohand transform on both side in equation (6), we have

$$M\{y''\} - 3M\{y'\} + 2M(y) = 0$$

$$v^2 M(y) - v^3 y(0) - v^2 y'(0) - 3vM(y) + 3v^2 y(0) + 2M(y) = 0$$

$$(v^2 - 3v + 2)M(y) = v^3 + v^2$$

$$M(y) = \frac{v^3 + v^2}{v^2 - 3v + 2}$$

$$M(y) = \frac{3v^2}{v - 2} - \frac{2v^2}{v - 1}$$

The inverse Mohand transform of the equation above gives us the solution:

$$y(t) = 3e^{2t} - 2\, e^{t}$$

5. Conclusions

In the present article, we have defined Mohand transform to solve linear ordinary differential equations with constant coefficients. We can say that the proposed transform method is, as an alternative approach, easy and understandable. Examples show that the method can be applied without longer calculations. The examples here can also be solved with other integral transforms available in the literature. We preferred this transform method because we think that not much work has been done on this subject. However, when the Mohand transform is compared with the Laplace transform, it can be seen that both methods work in the same way and give the exact solution of ordinary differential equations. This comparison will be the subject of our another study. I think that different useful integral transforms will also be mentioned by researchers in the future.

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