

Parametric Examination Anisotropic Thermal Resistance of MIL Composites

Kubra SOLAK^{1, *} ^(D), Resat MUTLU²

¹ Department of Mechanical Engineering, Tekirdağ Namık Kemal University, 59860, Tekirdağ, Turkey
²Department of Electronics and Telecommunication Engineering, Tekirdağ Namık Kemal University, 59860, Tekirdağ, Turkey

Highlights

- Anisotropic thermal resistance of MIL composites is studied using thermal and electrical circuit analogy.
- Comparison of the thermal resistances of MIL composites for different directions are made.
- A parametric analysis of the thermal resistances is performed.
- The findings may be used to make MIL composites with the desired thermal resistances.

Article Info	Abstract
Received: 27 May 2022 Accepted: 20 Oct 2022	Metallic-intermetallic laminate (MIL) composites possess intermediary properties emerging from the different laminates used. They are anisotropic since their properties are direction-dependent. The laminates used in a MIL composite have different thermal conductivities and this results in anisotropic thermal resistance. In a recent study, using the composite dimensions and the
Keywords MIL composites Thermal conductivity Thermal-electrical analogy, Anisotropic materials	electrical conductivity of the laminates used to make the MIL composite, the electrical resistance of rectangular prism-shaped MIL composites for different directions is examined. Since thermal and electrical circuits are analogs, a similar analysis can also be done for thermal conduction quantities. In this study, using the composite dimensions and the thermal conductivity of the laminates used to make the MIL composite, the thermal resistance of rectangular prism-shaped MIL composite, the thermal resistance of rectangular prism-shaped MIL composites for different directions is calculated and its direction-dependent parametric examination is carried out.

1. INTRODUCTION

Composite materials are made by combining the good properties of two or more constituent materials in a single material and by combining them at a macroscopic level to obtain new features such as tensile strength, compression, bending, yield, creep, fatigue strength, hardness, toughness, stiffness, abrasion resistance, electrical conductance, and thermal conductance [1,2]. Composite materials are grouped as fiber-reinforced, particulate, laminated, and compound composites according to their production method [1,2]. Metallic-intermetallic laminate (MIL) composites are an important member of the composite family. MIL composites are primarily made by combining two different metals in a thin foil structure in a laminated structure and by exposing them to the required sintering processes. Such composites unite the ductility of the metallic phase and the hardness of the intermetallic phase without the brittleness of the intermetallics [3-8]. The electrical conductivity of composite materials is difficult to model and its modeling may need tensors [9-13]. Anisotropic materials may have directionally varying electrical properties such as conductivity, permittivity, and permeability [1,2-14]. Some materials have anisotropic thermal conductivity [15-19]. There is an analogy between thermal and electrical circuits [20,21]. In [22], it is shown that such an analogy can be used in modeling transient heat flow. Such analogies are used in the modeling, analysis, and control of thermal systems [23-27]. Modeling of such thermal resistances is also important for the thermal dynamics of the electric current-activated/assisted sintering systems [28-30]. The directional dependence of the electrical resistance of the MIL composites has been examined in [31]. The calculation method of thermal resistance is similar to that of electrical resistance [20,21]. A researcher who needs a thermal resistance formula may often find the formula needed in an electromagnetic theory book [20,21]. To the best of our knowledge, the directional dependence of thermal resistance of the MIL composites has not been inspected in the literature yet. Such an analysis can be done considering the two cases: the laminates are connected in series and in parallel as done in [31]. The purpose of the study is to determine the thermal properties of rectangular prism-shaped MIL composite materials. The dimensions and the thermal conductivity of the laminates of the rectangular prism-shaped MIL composite are used to find its anisotropic equivalent resistances. Direction-dependency of the thermal resistances of a MIL composite is examined using the formulas parametrically.

The study is organized as follows. Analogies between thermal and electrical circuits are briefly explained in the second section. Anisotropic thermal resistances of MIL composites are determined and analyzed parametrically in the third section. A comparison of thermal resistances of MIL composites for different directions is made in the fourth section. The article is finished with a conclusion section.

2. ELECTRICAL ANALOGIES BETWEEN THERMAL AND ELECTRICAL CIRCUITS

There is a useful analogy between thermal and electrical circuits [20]. It is well-known [32] that Ohm's Law is given as

$$I = \frac{V}{R}.$$
(1)

The electrical resistance of a slab with a uniform cross-section is calculated as:

$$R = \frac{l}{\sigma A} \tag{2}$$

where σ is the electrical conductivity, A is the resistor area perpendicular to the current density, and l is the length of the resistor.

Fourier's law [21] can be stated as:

$$q = \frac{\Delta T}{R_{th}} \tag{3}$$

where q is the local heat flux density, W/m². ΔT is the temperature difference, and R_{th} is the thermal resistance.

The thermal resistance of a slab with a uniform cross-section is calculated as:

$$R_{th} = \frac{l}{kA} \tag{4}$$

where k is the thermal conductivity, A is the thermal resistor area perpendicular to the heat flux density, and l is the thermal resistor length.

As seen from the equations, Equation (1) is similar to Equation (3), i.e., Fourier's law is analogous to Ohm's law [20,21] and Equation (2) is similar to Equation (4). The combination rules for any number of resistors in series or parallel can be derived with the use of Ohm's Law, the voltage law, and the current law. That's why series and parallel equivalent resistance formulas of the electrical circuits can also be used to calculate series and parallel equivalent thermal resistances [20-22].

A series resistor circuit in which there are N series-connected resistors is shown in Figure 1.



The current is the same in each series-connected resistor by Kirchhoff's current law and the total voltage of the resistors is the sum of the voltages across individual resistors Kirchhoff's voltage law [20]:

$$R_{equivalent} = \frac{V}{I} = \frac{V_1 + V_2 + V_3 + \dots + V_N}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \dots + \frac{V_N}{I_N}$$
(5)

The series electrical resistance formula is given as

$$R_{eauivalent} = R_1 + R_2 + R_3 + \dots + R_N . (6)$$

A parallel resistor circuit in which there are N parallel-connected resistors is shown in Figure 2.



Figure 2. A parallel resistor circuit

The voltage is the same across each parallel-connected resistor by Kirchhoff's voltage law and the total current of the resistors is the sum of the currents flowing through the individual resistors by Kirchhoff's current law [20]:

$$\frac{V}{R_{equivalent}} = I = I_1 + I_2 + I_3 + \dots + I_N = \frac{V_1}{I_1} + \frac{V_2}{I_2} + \frac{V_3}{I_3} + \dots + \frac{V_N}{I_N}$$
(7)

The electrical resistance formula of the parallel-connected resistors is given as

$$R_{equivalent} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}}$$
(8)

The formulas can also be used for the thermal resistances of the MIL composites due to the thermal and electrical analogy and they are to be used in the next section to calculate the direction-dependent thermal resistances.

3. THERMAL RESISTANCE OF MIL COMPOSITES FOR DIFFERENT DIRECTIONS

Metallic-intermetallic laminate (MIL) composites are an important member of the composite family. MIL composites are primarily made by combining two different metals in a thin foil structure in a laminated structure and subjecting them to the necessary sintering processes as shown in Figure 3.



Figure 3. Structure of the MIL composite

A schematic presentation of layering and synthesis of the constituent materials can be seen in Figure 3. The constituent materials are pressurized at a sufficiently high temperature using the apparatus shown in Figure 4. An SEM micrograph which is acquired at the end of the production process for a typical MIL composite is given in Figure 5.



Figure 4. The mechanical design used to synthesize the array [31]



Figure 5. SEM micrograph of a MIL Composite [31]

A rectangular prism-shaped MIL composite whose anisotropic electrical conductivity is examined in [31] is shown in Figure 6. If it is assumed that the MIL composite is in thermal equilibrium and there is a temperature difference across two opposite faces of the prism, the opposite faces where heat flux enters and exists are equipotential surfaces. The thermal resistance between the left and right faces of the prism as shown in Figure 6 is called $R_{th,side}$ which is also the equivalent thermal resistance of the parallel-connected laminates. The thermal resistance between the up and down faces of the prism as shown in Figure 6 is called $R_{th,side}$ which is also the equivalent thermal resistance of the parallel-connected laminates. The thermal resistance between the up and down faces of the prism as shown in Figure 6 is called $R_{th,up}$ and which is also the equivalent thermal resistance of the series-connected laminates. The thermal resistance between the prism is not examined in this paper. By switching a with L, the $R_{th,side}$ formula can be used to calculate it if desired.



Figure 6. Representation of $R_{th,up}$ and $R_{th,side}$ thermal resistances, and schematic structure of the dimensions of the MIL composite

If the heat flux enters from the left face and leaves the right face or vice versa, its equivalent thermal resistance is found as the equivalent thermal resistance of the parallel laminates using Equation (8);

$$\frac{1}{R_{th,side}} = \sum_{i=1}^{i=N} \frac{1}{R_{th,i}} = \sum_{i=1}^{i=N} \frac{1}{\frac{L}{k_i b_i a}} = \frac{a}{L} \sum_{i=1}^{i=N} k_i b_i$$
(9)

$$R_{th,side} = \frac{L}{a} \frac{1}{\sum_{i=1}^{i=N} k_i b_i}$$
(10)

$$R_{th,side} = \left(\frac{L}{a}\right) \frac{1}{\left(N_e k_e b_e + N_o k_o b_o\right)} \tag{11}$$

where the number of odd layers is N_o , the number of even layers is N_e , the thickness of odd-numbered layers is b_o , thickness of even-numbered layers is b_e , k_o is the thermal conductance of odd-numbered layers, and k_e is the thermal conductance of even layers.

The total number of the laminates, N, is calculated as

$$N = N_e + N_o \quad . \tag{12}$$

If the heat flux enters from the up face and leaves the down face or vice versa, its thermal resistance is calculated as the equivalent thermal resistance of the series laminates using Equation (6);

$$R_{th,up} = R_{th,down} = \sum_{i=1}^{i=N} \frac{b_i}{k_i L a} = \frac{1}{L a} \sum_{i=1}^{i=N} \frac{b_i}{k_i}$$
(13)

$$R_{th,up} = \frac{1}{La} \left(\frac{N_e b_e}{k_e} + \frac{N_o b_o}{k_o} \right). \tag{14}$$

4. COMPARISON OF THERMAL RESISTANCES OF MIL COMPOSITES FOR DIFFERENT DIRECTIONS

In this section, the ratio of the laminates' thermal conductivities, and the ratio of the thermal resistance of series connected laminates to the thermal resistance of parallel connected laminates are inspected regarding the prism-shaped MIL geometry. Let's arrange Equation (11) as

$$R_{th,side} = \frac{L}{a} \frac{1}{\left(N_e b_e + N_o b_o \frac{k_o}{k_e}\right)k_e}$$
(15)

Let's designate the ratio of the MIL laminates' thermal conductivities as

$$\phi = \frac{k_o}{k_e} \quad . \tag{16}$$

Then, Equation (15) becomes

$$R_{th,side} = \frac{L}{a} \frac{1}{(N_e b_e + N_o b_o \emptyset)k_e}$$
(17)

Let's arrange Equation (14) using \emptyset as

$$R_{th,up} = \frac{1}{Lak_e} \left(N_e b_e + \frac{N_o b_o}{\frac{k_o}{k_e}} \right) = \frac{1}{Lak_e} \left(N_e b_e + \frac{N_o b_o}{\emptyset} \right).$$
(18)

The ratio of the thermal resistance of series connected laminates to the thermal resistance of parallel connected laminates is found as

$$\frac{R_{th,up}}{R_{th,side}} = \frac{\left(N_e b_e + \frac{N_o b_o}{\emptyset}\right) (N_e b_e + N_o b_o \emptyset)}{L^2}$$
(19)

$$\frac{R_{th,up}}{R_{th,side}} = \frac{(N_e b_e)^2 + (N_o b_o)^2 + (N_e b_e)(N_o b_o)\left(\emptyset + \frac{1}{\emptyset}\right)}{L^2}.$$
(20)

 $R_{th,up}$ and $R_{th,side}$ thermal resistance with respect to the thermal conductivity ratio of the layers are shown in Figures 7 and 8. Their thermal resistance ratio with respect to the thermal conductivity ratio is shown in Figure 9.



Figure 7. $R_{th,up}$ thermal resistance with respect to the thermal conductivity ratio of the layers of the laminate for fixed k_o



Figure 8. $R_{th,side}$ thermal resistance with respect to the thermal conductivity ratio of the layers for fixed k_o



Figure 9. Thermal resistance ratio with respect to the thermal conductivity ratio of the layers

 $R_{th,up}$ and $R_{th,side}$ thermal resistances with respect to the layer thickness ratio are shown in Figures 10 and 11.



Figure 10. R_{th,up} thermal resistance with respect to the thickness ratio of the layers



Figure 11. R_{th,side} thermal resistance with respect to the thickness ratio of the layers

Equation (20) shows that when the length of the prism-shaped MIL increases, $R_{th,side}$ becomes negligible when compared to $R_{th,up}$.

When
$$N_o$$
 is high,

$$N = N_c \cong N_c \tag{21}$$

and it can be assumed that

$$N \cong 2N_e \text{ or } N \cong 2N_o \quad . \tag{22}$$

Then, Equation (20) turns into

$$\frac{R_{th,up}}{R_{th,side}} = (N_e)^2 \frac{(b_e)^2 + (b_o)^2 + (b_e)(b_o)\left(\emptyset + \frac{1}{\emptyset}\right)}{L^2}$$
(23)

$$\frac{R_{th,up}}{R_{th,side}} = \left(\frac{N}{2}\right)^2 \frac{(b_e)^2 + (b_o)^2 + (b_e)(b_o)\left(\phi + \frac{1}{\phi}\right)}{L^2} \quad .$$
(24)

If the thicknesses are equal,

$$b_e = b_o \ . \tag{25}$$

Further reduction of Equation (24) is possible;

$$\frac{R_{th,up}}{R_{th,side}} = \frac{\left(\frac{Nb_e}{2}\right)^2 \left(2 + \emptyset + \frac{1}{\emptyset}\right)}{L^2} \quad . \tag{26}$$

As it can be seen from Figure 6, the following is true;

$$b = Nb_e \,. \tag{27}$$

Then,

$$\frac{R_{th,up}}{R_{th,side}} = \frac{\left(\frac{b}{2}\right)^2 \left(2 + \phi + \frac{1}{\phi}\right)}{L^2}$$
(28)

$$\frac{R_{th,up}}{R_{th,side}} = \frac{\left(\frac{b}{L}\right)^2 \left(2 + \phi + \frac{1}{\phi}\right)}{4} . \tag{29}$$

If the laminates have the same thermal conductivities,

$$\phi = 1 \tag{30}$$

$$\frac{R_{th,up}}{R_{th,side}} \cong \left(\frac{b}{L}\right)^2.$$
(31)

Equation (29) is normalized $\left(\frac{b}{L}\right)^2$ by plotted and shown in Figure 12. If $\emptyset \ge 1$ or $\emptyset \le 1$, $\frac{R_{th,up}}{R_{th,side}} \left(\frac{L}{b}\right)^2$ becomes quite higher than one (1).



5. CONCLUSION

Some MIL composites can be used for electromagnetic shielding in military applications due to their direction-dependent electrical resistance. However, there is also a need for understanding the direction-dependent thermal behavior of these kinds of MIL composites. There may be a thermal discomfort or a trade-off resulting from their usage. In order to make the best use of the MIL composites, their thermal behavior must also be examined. The directional dependence of the thermal resistance of the MIL composites to prevent undesired thermal behavior. In this study, using parameters, the direction-dependent thermal resistances of a rectangular prism-shaped MIL composite are calculated. Using the ratio of the laminates' thermal conductivities, the ratio of the thermal resistance of series connected laminates to the thermal resistance of parallel connected laminates is examined.

The analysis shows that the equivalent thermal resistance of the series connected layers of the MIL composite, $R_{th,up}$ increases with respect to the thermal conductivity ratio between intermetallic and metallic layers as shown in Figure 7. For $R_{th,side}$ thermal resistance, the MIL composite layers in the structure are considered in parallel and, in this case, as the thermal conductivity ratio increases, the equivalent thermal resistance of the parallel connected layers of the MIL composite, $R_{th,side}$, decreases, and it is almost determined by the metallic layer which has the higher thermal conductivity as shown in Figure 8. $R_{th,up}$ also increases as the thickness ratio of the layers increases as shown in Figure 10. $R_{th,side}$ also decreases as the thickness ratio of the layers increases as shown in Figure 11. It has been also found that the number of intermetallic (N_e) and metallic (N_o) layers can be considered equal for high N values without any significant error.

Consequently, this study has also shown that the possibility of maximizing the thermal conductance of MIL composites along a desired axis is achievable by playing physical dimensions. Results obtained in this paper may provide significant opportunities for obtaining MIL composites with the desired thermal conductivity behavior. It is expensive to make experiments to prepare the MIL composites and sometimes trial and error methods are used to prepare the samples which cause an increase in the cost and the time needed for their production. The formulas given here can be used to decrease the number of experiments made to obtain the desired thermal behavior of the MIL composites more economically. As future work, we suggest analyzing the steady-state and transient thermal behaviors of the MIL composites with and without an electrical current under different boundary conditions.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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