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Research Article

Adomian Decomposition and Variational Iteration Methods in the Context of Partial Differential Equations

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ABSTRACT

Partial Differential Equations (PDE) help model problems in science and engineering, given their abilities to capture complex phenomena compared to Ordinary Differential Equations. This paper aims to investigate two semi-analytical techniques called the Adomian Decomposition Method (ADM) and the Variational Iteration Method (VIM), how these methods can be used in practice, and to make a comparative study of the two methods for solving linear and nonlinear PDEs. The efficiency of ADM and VIM is assessed by comparing their errors relative to the exact solutions of the examined numerical experiments. The results obtained from the numerical experiments revealed that ADM proved to be a more efficient and accurate method for solving PDEs than VIM.

1. Introduction

In physics and engineering, partial differential equations (PDEs) are utilized to model diverse problems, given their ability to describe the change of a system concerning multiple independent variables. Investigating solutions of differential equations has been significant to scientists and researchers [14]. Given the challenges faced in obtaining analytical solutions for PDEs, especially those with non-linearity, numerical methods have been employed to address these problems [18,20]. Nevertheless, despite the various numerical methods for solving differential equations, significant limitations remain in numerical analysis for tackling PDE problems [10,11]. Some works on semi-analytical methods have been proposed in extant literature to approximate solutions to such problems [1,14,20,27,28].

Considering the Adomian Decomposition Method (ADM), George Adomian was the first to begin using ADM to find the solution of functions by illustrating them in the form of a series. This method uses an iterative formula to calculate subsequent series parts based on initial and boundary conditions. Wazwaz and El-Sayed further re ned ADM to address this, enabling rapid convergence and easily computable components. Researchers have developed modified techniques to enhance ADM's efficiency. For instance, a new approach was proposed to solve time-fractional diffusion equations with initial and boundary conditions, Volterra-Fredohlm integral-equations, demonstrating improved accuracy and convergence [4.21]. Additionally, ADM has been effectively applied to systems of second-order differential-algebraic equations and systems of PDEs, providing approximate solutions that align closely with exact results [6,24]. Meanwhile, He and Wu developed the Variational Iteration Method (VIM), which involves constructing a correction function related to the solved equation, incorporating a Lagrange multiplier [2,12].

The VIM is a highly effective technique commonly employed in analyzing various mathematical models. Unlike other methods, it does not necessitate any transformation of the given problem, allowing for direct application similar to the ADM. Implementing VIM for various types of linear and nonlinear partial differential equations was carried out by Shihab et al. [25]. Fatima et al. [15] used the VIM to solve nonlinear partial differential equations in fluid dynamics. At the same time, authors [5, 23,30] utilized a blend of VIM with the Sumudu transform to resolve some nonlinear equations. Recent studies [1,12,13,17,19,22] have explored the efficacy of the ADM and the VIM in solving specific complex differential equations. Comparative studies have also been conducted to evaluate the effectiveness of ADM and VIM in solving nonlinear differential equations, providing insights into their respective advantages and applications [7.8,16,29, 31,32]. This work compares ADM and VIM methods to solve both linear and nonlinear partial differential equations. The methods, their numerical solution, and errors will be analyzed to aid understanding of the application of both methods to solve PDE problems. Novel contributions include a comparative analysis of the ADM and the VIM in solving linear and nonlinear PDEs, which introduces new understanding and insights into the suitability of the two methods for different types of PDE problems and bridges research gaps in the literature.

The subsequent section of this work is segmented into four (4) parts. The first part provides an overview of the ADM and the VIM development for both linear and nonlinear PDEs. The second part declares the numerical experiments via which the methods' efficiency and accuracy are evaluated and the results of the numerical experiment after computations have been carried out. Following this, the third part discusses the results that were attained to provide context for their implications in applications. Lastly, a conclusion is made, and recommendations are proposed for researchers seeking to improve this body of work in the last part.

2. Material and Methods

2.1. Description of Adomian Decomposition Method (ADM)

The study Given the general differential equation in the form:

$$Fv(u,t) = g(u,t) \tag{1}$$

With conditions (initial) $v(u,0) = f(u)$, where F denotes a differential operator involving nonlinear and linear terms. Hence, Equation (1) can take the form

$$L_t v(u,t) + Rv(u,t) + Nv(u,t) = g(u,t) \tag{2}$$

where $L_t = \frac{\partial}{\partial t}$, R denotes a linear operator containing partial derivatives with respect to u,N represents a nonlinear operator and g is the source term or a nonhomogenous term that is independent of v. Algebraically,

$$L_t v(u,t) = g(u,t) - Rv(u,t) - Nv(u,t) \tag{3}$$

Inversion of the operator L_t, (L_t^{-1}) and application of it to both sides of equation (3) gives

$$L_t^{-1} L_t v(u,t) = L_t^{-1} g(u,t) - L_t^{-1} Rv(u,t) - L_t^{-1} Nv(u,t) \tag{4}$$

Which results into

$$v(u,t) = f(u) + L_t^{-1}g(u,t) - L_t^{-1}Rv(u,t) - L_t^{-1}Nv(u,t) \quad (5)$$

where f(u) denotes constant of integration concerning that satisfies $L_t f=0$. The unknown function v(u, t) is decomposed into an infinite series as:

$$v(u,t) = \sum_{p=0}^{\infty} v_p(u,t) \quad (6)$$

The nonlinear operator N(v) is decomposed as:

$$Nv(u,t) = \sum_{p=0}^{\infty} A_p(v_0,\ v_1\ v_2, \cdots, v_p) \quad (7)$$

where sequence $[A_p]_{p=0}^{\infty}$ is called the Adomian polynomial sequence and explicit computation of the nonlinear A_p terms is given by

$$A_0(v_0) = N(v_0)$$
$$A_1(v_0,\ v_1) = N'(v_0)v_1$$
$$A_2(v_0,\ v_1,\ v_2) = N'(v_0)v_2 + \frac{{v_1}^2}{2!}N''(v_0)$$
$$A_3(v_0,\ v_1,\ v_2,\ v_3) = N'(v_0)v_3 + N''(v_0)_{12}\frac{{v_1}^3}{3!}''(v_0) \vdots$$

Hence the summarized format is given by the relation

$$A_p(v_0,\ v_1\ v_2, \cdots, v_p) = \frac{1}{p!}\frac{d^p}{d\beta^p}\left[N\left(\sum_{j=0}^{p}\beta^j v_j\right)\right]_{\beta=0} \quad (8)$$

Thus, substituting equations (6), (7) and (8) into (5) results into

$$\sum_{p}^{\infty} v_p(u,t) = f(u) + L_t^{-1}g(u,t) - L_t^{-1}R\sum_{p}^{\infty} v_p(u,t) - L_t^{-1}\sum_{p=0}^{\infty} A_p(v_0,\ v_1\ v_2, \cdots, v_p) \quad (9)$$

Identification of v_0 as $f(u) + L_t^{-1}g(u,t)$, it is possible to write

$$\vdots$$
$$v_{p+1}(u,t) = -L_t^{-1}Rv_p(u,t) - L_t^{-1}A_p(v_0,\ \cdots\ ,\ v_p) \quad (10)$$

By isolating the linear and nonlinear components and equating terms having the same order, the resulting recursive algorithm becomes

$$\begin{cases} v_0(u,t) = f(u) + L_t^{-1}g(u,t) \\ v_{p+1}(u,t) = L_t^{-1}Rv_p(u,t) \\ \quad -L_t^{-1}A_p(v_0,\ v_1,\ \cdots,\ v_p) \end{cases}, \quad p = 0,\ 1,\ 2,\ 3,\ \cdots \quad (11)$$

Using the recursive algorithm defined in Equation (11), an approximate solution to Equation (1) can be obtained through a series expansion.

$$v_j(u,t) = \sum_{p=0}^{j} v_p(u,t)$$

where

$$\lim_{j\to\infty}\sum_{p=0}^{j} v_p(u,t) = v(u,t) \quad (12)$$

Given the right conditions, the series $\sum_{p=0}^{\infty} v_p(u,t)$ converges to the solution v(u,t) of the initial problem. The decomposition of the series solution tends to converge rapidly, requiring only a few terms for effective solution analysis. The conditions governing this convergence have been extensively studied in references [8,17,21,22].

2.2. Description of Variational Iteration Method (VIM)

Consider the following general nonlinear differential equation:

$$Lv(u,t) + Rv(u,t) + Nv(u,t) = g(u,t) \quad (13)$$

where L denotes a linear operator, N represents a nonlinear operator, v(u,t) is a known function, and g(u,t) is a known analytical function. The correction functional can be constructed thus

$$v_{p+1}(u,t) = v_p(u,t) + \int_0^t \lambda(\eta) [Lv(u,\eta) + N\tilde{v}(u,\eta) - g(u,\eta)]d\eta \quad , \quad p \geq 0 \tag{14}$$

where $\lambda(\eta)$ denotes the Lagrange multiplier, which can be determined optimally using the variational theory. The subscript p refers to the p^{th} approximation and $\tilde{v}_p$ is considered a restricted variation, meaning $\delta\tilde{v}_p = 0$. By first determining the Lagrange multiplier λ through integration by parts, the successive approximation $v_{p+1}, p \geq 0$, can be obtained using the selected Lagrange multiplier and any initial function v_0. It can be obtained through the stationary functions

$$\begin{aligned} 1 + \lambda|\eta = t = 0 \\ \lambda'|\eta = t = 0 \end{aligned} \tag{15}$$

where one can find the next Lagrange multiplier as follows

$$\begin{aligned} \lambda = -1 \quad for \quad r = 1 \\ \lambda = \eta - t \; for \quad r = 2 \end{aligned} \tag{16}$$

In addition, the standard formula for the Lagrange multiplier in the scenario $r \geq 1$ is denoted as

$$\lambda(\eta) = \frac{(-1)^r(\eta - t)^{r-1}}{(r-1)!} \tag{17}$$

After determining the value of $\lambda(\eta)$, substituting it into the corrective function in equation (14) enables us to derive the following iteration formula.

$$v_{p+1}(u,t) = v_p(u,t) + \int_0^t \frac{(-1)^r(\phi - t)^{r-1}}{(r-1)!} [Lv(u,\eta) + N\tilde{v}(u,\eta) - g(u,\eta)]d\eta \tag{18}$$

For $\lambda = -1$, the iteration formula becomes:

$$v_{p+1}(u,t) = v_p(u,t) - \int_0^t [Lv(u,\eta) + N\tilde{v}(u,\eta) - g(u,\eta)]d\eta \tag{19}$$

Applying the iterative formula in equations (18) or (19) yields the sequence from a suitable initial guess. Advanced computing allows repeated iterations until the desired precision is achieved. The approximate solution is then given by

$$v(u,t) = \lim_{p\to\infty} v_p(u,t) \tag{20}$$

3. Numerical Experiments

The two methods are applied to a range of linear and nonlinear PDEs with known solutions to evaluate the accuracy of the VIM and ADM methods. The methods' performance is assessed by comparing computed results obtained using Python's Jupyter Notebook 2022 to analytical solutions. The findings are presented in Tables 1-6 and illustrated in Figures 1-3, highlighting the computed solutions and corresponding errors.

Experiment 1: Using ADM and VIM, solve the following linear PDE

$$au_a + u_y = 3u \tag{21}$$

with the following initial conditions:

$u(a,0) = a^2, u(0,y) = 0$, and analytical solution: $u(a,y) = a^2 l^y$

ADM Solution: Re-write (21) in an operator form as

$$L_y u(a,y) = 3u(a,y) - aL_a u(a,y) \tag{22}$$

The inverse operator is applied to both sides of (22) alongside the given condition $u(a,0) = a^2$ results in

$$u(a,y) = a^2 + L_y^{-1}(3u - aL_a u) \tag{23}$$

Substitute $u(a,y) = \sum_{p=0}^{n} u_p(a,y)$ into both sides of (23)

$$u_p(a,y) = a^2 + L_y^{-1} \sum_{p=0}^{\infty} \left(3\left(\sum_{p=0}^{\infty} u_p(a,y)\right) - aL_a\left(\sum_{p=0}^{\infty} u_p(a,y)\right) \right) \tag{24}$$

Taking few components of the decomposition of $u(a,y)$, equation (24) becomes

$$u_0 + u_1 + u_2 + \cdots = a^2 + L_y^{-1}\begin{pmatrix} 3(u_0 + u_1 + u_2 + \cdots) - \\ aL_a(u_0 + u_1 + u_2 + \cdots) \end{pmatrix} \quad (25)$$

The recursive terms are identified

$$u_o(a,y) = a^2$$

$$u_{q+1}(a,y) = L_y^{-1}\left(3a_q - aL_a u_q\right), \quad q = 0 \quad (26)$$

The first four components are obtained thus

$$u_0(a,y) = a^2$$
$$u_1(a,y) = L_y^{-1}(3u_0 - aL_a u_0) = a^2 y,$$

$$u_2(a,y) = L_y^{-1}(3u_1 - aL_a u_1) = \frac{a^2y^2}{2!},$$

$$u_3(a,y) = L_y^{-1}(3u_2 - aL_a u_2) = \frac{a^2y^3}{3!}, \quad (27)$$

VIM Solution: The correction functional is constructed as

$$u_{p+1}(a,y) = u_p(a,y) + \int_0^y \lambda(\eta)\begin{pmatrix} \dfrac{\partial u_p(a,\eta)}{\partial \eta} + \\ a\dfrac{\partial \tilde{u}_p(a,\eta)}{\partial a} - 3\tilde{u}_p(a,\eta) \end{pmatrix} d\eta \quad (28)$$

The stationary conditions are

$$1 + \lambda|\eta = a = 0$$

$$\lambda'|\eta = a = 0 \quad (29)$$

which results in

$$\lambda = 1 \quad (30)$$

Substituting the Lagrange multiplier (30) into (28), the following iteration formula is obtained

$$u_{p+1}(a,y) = u_p(a,y) - \int_0^y \begin{pmatrix} \dfrac{\partial u_p(\eta,y)}{\partial \eta} \\ +a\dfrac{\partial u_p(\eta,y)}{\partial a} - 3u_p \end{pmatrix} d\eta, \quad p \geq 0 \quad (31)$$

Selecting $u_0(a,y) = a^2$ from the given conditions and substitute it into (31) results in the first four successive approximations as follows

$$u_0(a,y) = a^2$$
$$u_1(a,y) = a^2 - \int_0^y \left(\frac{\partial u_0(a,\eta)}{\partial \eta} + a\frac{\partial u_0(a,\eta)}{\partial a} - 3u_0(a,\eta)\right) d\eta = a^2 + a^2 y,$$
$$u_2(a,y) = a^2 + a^2 y - \int_0^y \left(\frac{\partial u_1(a,\eta)}{\partial \eta} + a\frac{\partial u_1(a,\eta)}{\partial a} - 3u_1(a,\eta)\right) d\eta = a^2 + a^2 y + \frac{1}{2!}a^2y^2,$$

$$u_3(a,y) = a^2 + a^2 y + \frac{1}{2!}a^2y^2 - \int_0^y \begin{pmatrix} \dfrac{\partial u_2(a,\eta)}{\partial \eta} + \\ a\dfrac{\partial u_2(a,\eta)}{\partial a} - 3u_2(a,\eta) \end{pmatrix} d\eta = a^2 + a^2 y + \frac{1}{2!}a^2y^2 + \frac{1}{3!}a^2y^2, \quad (32)$$

Experiment 2: Compute the nonlinear PDE by applying ADM and VIM

$$u_y + uu_a = 0 \quad (33)$$

given the following initial condition:

$$u(a,0) = a \quad y > 0$$
$$where\ u = u(a,y)$$

and analytical solution: $u(a,y) = \dfrac{a}{1+y}, \ |y| < 1$

ADM Solution: Re-write (33) in an operator form as

$$L_y u(a,y) = -uu_a \quad (34)$$

where $L_y$is defined using

$$L_y = \frac{\partial}{\partial y} \quad (35)$$

The inverse operator L_y^{-1} is added to both sides of (34) with the initial condition to obtain

$$u(a,y) = a - L_y^{-1} u u_a \tag{36}$$

Substituting

$$u(a,y) = \sum_{p=0}^{\infty} u_p(a,y) \tag{37}$$

and the nonlinear term

$$u u_a = \sum_{p=0}^{\infty} u_p(a,y) \tag{38}$$

into equation (34) results in

$$\sum_{p=0}^{\infty} u_p\,(a,y) = a - L_y^{-1}\left(\sum_{p=0}^{\infty} A_p\right) \tag{39}$$

hence, the recursive relation is obtained as

$$u_0(a,y) = a,$$

$$u_{q+1}(a,y) = -L_y^{-1}\left(A_q\right), \quad q \geq 0 \tag{40}$$

Thus, the result of the first four components are as follows

$$\begin{aligned} &u_0(a,y) = a \\ &u_1(a,y) = -L_y^{-1}A_0 = -L_y^{-1}(a) = -ay \\ &u_2(a,y) = -L_y^{-1}A_1 = -L_y^{-1}(-2ay) = ay^2 \\ &u_3(a,y) = -L_y^{-1}A_2 = -L_y^{-1}(3ay^2) = -ay^3 \end{aligned}$$

VIM Solution: The correction functional for (33) is given by

$$u_{p+1}(a,y) = u_p(a,y) + \int_0^y \lambda(\eta)\left(\frac{\partial u_p(a,\eta)}{\partial \eta} + \tilde{u}_p(a,\eta)\frac{\partial \tilde{u}_p(a,\eta)}{\partial a}\right) d\eta \tag{41}$$

and the stationary conditions

$$1 + \lambda | \eta = y = 0$$

$$\lambda' | \eta = y = 0 \tag{42}$$

gives

$$\lambda = -1 \tag{43}$$

To obtain the iteration formula, the Lagrange multiplier $\lambda = -1$ is substituted into the functional (41) as

$$u_{p+1}(a,y) = u_p(a,y) - \int_o^y \left(\frac{\partial u_p(a,\eta)}{\partial \eta} + u_p(a,\eta)\frac{\partial u_p(a,\eta)}{\partial a}\right) d\eta, \quad p \geq 0 \tag{44}$$

The first four successive approximations obtained by selecting $u_0(a,y) = a$ from the given initial condition are as follows

$$\begin{aligned} &u_0(a,y) = a, \\ &u_1(a,y) = a - ay, \\ &u_2(a,y) = a - ay + ay^2 - \frac{1}{3}ay^3, \\ &u_3(a,y) = a - ay + ay^2 - ay^3 + \frac{2}{3}ay^4 \end{aligned}$$

Experiment 3: Resolve the nonlinear PDE using ADM and VIM

$$u_y = a^2 + \frac{1}{4}u_a^2 \tag{45}$$

given the following initial condition:

$$u(a,0) = 0$$
$$where\; u = u(a,y)$$

and analytical solution: $u(a,y) = a^2 \tan y$

ADM Solution: Re-write (45) in an operator form as

$$u(a,y) = a^2 y + \frac{1}{4}L_y^{-1}u_a^2 \tag{46}$$

$u(a,y)$ is defined by

$$u(a,y) = \sum_{p=0}^{\infty} u_p\,(a,y) \tag{47}$$

having the nonlinear terms u_a^2 as

$$u_a^2 = \sum_{p=0}^{\infty} A_p \tag{48}$$

where A_p, $p \geq 0$ are the Adomian polynomials. Applying these assumptions yields

$$\sum_{p=0}^{\infty} u_p\,(a,y) = a^2 y + \frac{1}{4} L_y^{-1} \left(\sum_{p=0}^{\infty} A_p \right) \tag{49}$$

which results in the recursive relation

$$u_0(a,y) = u_{0_a}^2,$$

$$u_{q+1}(a,y) = \frac{1}{4} L_y^{-1} A_q, q \geq 0 \tag{50}$$

For this form of nonlinearity, the Adomian polynomials A_p are given by

$$A_0 = u_{0_a}^2,$$

$$A_1 = 2u_{0_a} u_{1_a},$$

$$A_2 = 2u_{0_a} u_{2_a} + u_{1_a}^2,$$

$$A_3 = 2u_{0_a} u_{3_a} + 2u_{1_a} u_{2_a} \tag{51}$$

and so on. The first four components are obtained as follows

$$\begin{aligned}
u_0(a,y) &= a^2 y, \\
u_1(a,y) &= \frac{1}{4} L_y^{-1} A_0 = \frac{1}{4} L_y^{-1} (4a^2 y^2) = \frac{1}{3} a^2 y^3, \\
u_2(a,y) &= \frac{1}{4} L_y^{-1} A_1 = \frac{1}{4} L_y^{-1} \left(\frac{8}{3} a^2 y^4 \right) = \frac{2}{13} a^2 y^5, \\
u_3(a,y) &= \frac{1}{4} L_y^{-1} A_2 = \frac{1}{4} L_y^{-1} \left(\frac{68}{45} a^2 y^6 \right) \\
&= \frac{17}{312} a^2 y^7
\end{aligned}$$

VIM Solution: Proceeding from the methods used in experiments 1 and 2, the correction function for the equation is

$$u_{p+1}(a,y) = u_p(a,y) - \int_0^y \lambda(\eta) \begin{pmatrix} \dfrac{\partial u_p(a,\eta)}{\partial \eta} \\ -\dfrac{1}{4} u_{p_2}^2(a,\eta) - a^2 \end{pmatrix} d\eta \tag{52}$$

The iteration formula is obtained as

$$u_{p+1}(a,y) = u_p(a,y) - \int_0^y \begin{pmatrix} \dfrac{\partial u_p(a,\eta)}{\partial \eta} \\ -\dfrac{1}{4} u_{p_2}^2(a,\eta) - a^2 \end{pmatrix} d\eta\,, \quad p \geq 0 \tag{53}$$

By using $u_0(a,y) = 0$ the given initial condition, the first four successive approximations are obtained as follows

$$\begin{aligned}
u_o(a,y) &= 0, \\
u_1(a,y) &= a^2 y \\
u_2(a,y) &= a^2 y + \frac{1}{3} a^2 y^3, \\
u_3(a,y) &= a^2 y + \frac{1}{3} a^2 y^3 + \frac{2}{15} a^2 y^5 + \frac{1}{63} a^2 y^7
\end{aligned}$$

The results of the experiments are computed in the following tables and figures.

Table 1. Computed Solution for Experiment 1

a/y	Solution of VIM	Solution of ADM	Analytical Solution
0.1	0.011051709180756	0.011051709180756	0.011051709180756
0.2	0.048856110326406	0.048856110326406	0.048856110326406
0.3	0.122487292681836	0.122487292681840	0.122487292681840
0.4	0.238691951622429	0.238691951622503	0.238691951622603
0.5	0.412180317671841	0.412180317672584	0.414280317675032
0.6	0.655962768106149	0.655962768140221	0.655962768140584
0.7	0.986737736402845	0.986738826635423	0.986738826660535
0.8	1.424346192769120	1.424246193675318	1.424346194235180
0.9	1.992278513156301	1.992278518003713	1.992278520037130
1.0	2.718281801146383	2.718281814564905	2.718281828459050

Table 1 presents the result of ADM and VIM numerical solution with the analytical solution for experiment 1.

Table 2. Comparison of Error for Experiment 1

a/y	Error of VIM	Error of ADM
0.1	0.000000000000000	0.000000000000000
0.2	0.000000000000000	0.000000000000000
0.3	0.000000000000004	0.000000000000000
0.4	0.000000000000174	0.000000000000100
0.5	0.000000000003191	0.000000000002448
0.6	0.000000000034435	0.000000000000363
0.7	0.000000009742310	0.000000000025112
0.8	0.000000001475060	0.000000000559862
0.9	0.000000006880829	0.000000002033417
1.0	0.000000027312667	0.000000013894145

Table 2 computes the comparison between the errors of ADM and VIM for experiment 1. The ADM is observed to exhibit lesser error than the VIM solution.

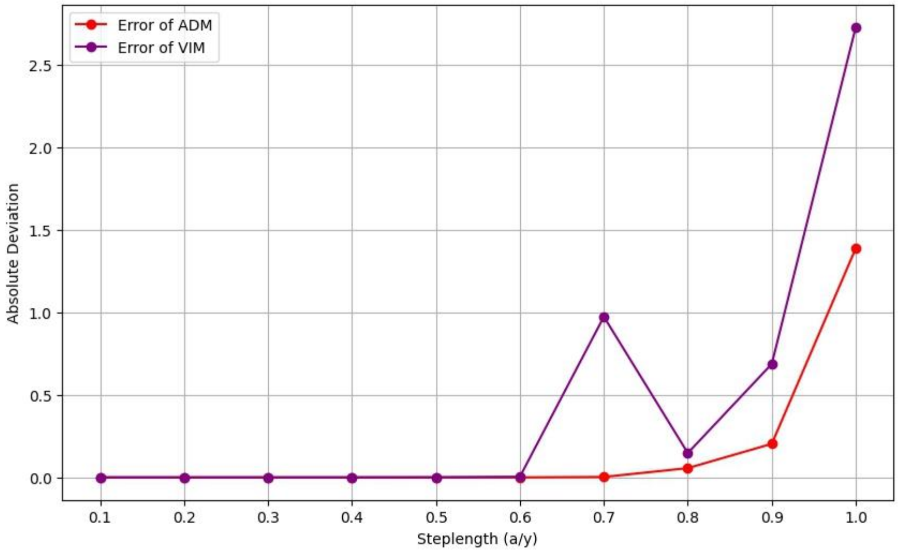

Figure 1. Error Plot for Experiment 1

Figure 1 illustrates the computed errors of ADM and VIM for experiment 1. The ADM is observed to exhibit a better performance than the VIM.

Table 3. Computed Solution for Experiment 2

a/y	Solution of VIM	Solution of ADM	Analytical Solution
0.1	0.087801242011922	0.090910000000000	0.090909090909091
0.2	0.175602484023843	0.181820000000000	0.181818181818182
0.3	0.263403726035765	0.272730000000000	0.272727272727273
0.4	0.351204968047687	0.363640000000000	0.363636363636364
0.5	0.429006210059608	0.454540000000000	0.454545454545455
0.6	0.526807452071530	0.545460000000000	0.545454545454545
0.7	0.614608694083452	0.636370000000000	0.636363636363636
0.8	0.702409936095374	0.727280000000000	0.727272727272727
0.9	0.790211178107295	0.818190000000000	0.818181818181818
1.0	0.878012420119217	0.909010000000000	0.909090909090909

Table 3 presents the result of ADM and VIM numerical solution with the analytical solution for experiment 2.

Table 4. Comparison of Error for Experiment 2

a/y	Error of VIM	Error of ADM
0.1	0.003107848897169	0.000000909090909
0.2	0.006215697794338	0.000001818181818
0.3	0.009323546691508	0.000002727272727
0.4	0.012431395588677	0.000003636363636
0.5	0.015539144485846	0.000004545454545
0.6	0.018647093383015	0.000005454545455
0.7	0.021754942280184	0.000006363636364
0.8	0.024862791177540	0.000007272727273
0.9	0.027970640074523	0.000008181818182
1.0	0.031078488971692	0.000009090909091

Table 4 computes the comparison between the errors of ADM and VIM for experiment 2. The errors of the ADM are lesser in contrast with the VIM errors.

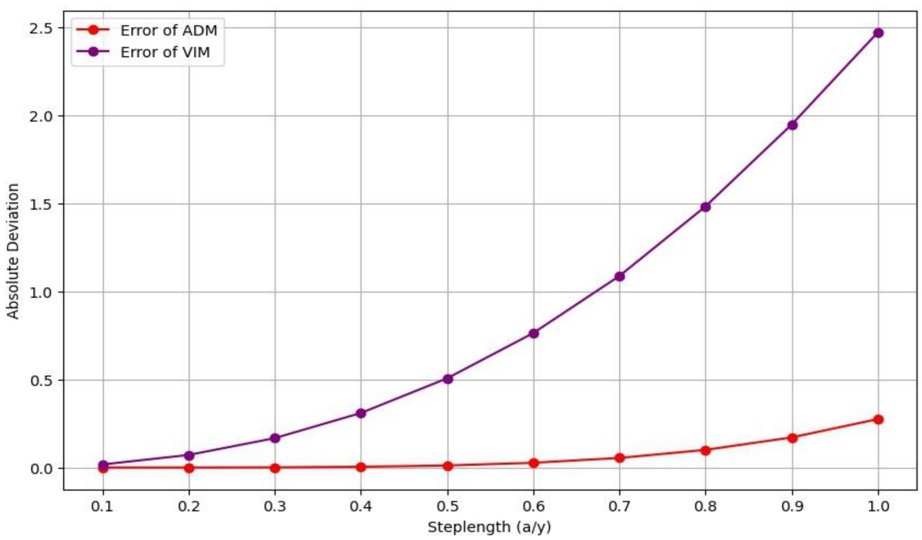

Figure 2. Error Plot for Experiment 2

Figure 2 illustrates the computed errors of ADM and VIM for experiment 2. The ADM is seen to have better performance than the VIM.

Table 5. Computed Solution for Experiment 3

a/y	Solution of VIM	Solution of ADM	Analytical Solution
0.1	0.009983300463875	0.009983341666667	0.010000000000000
0.2	0.047121963272002	0.047214724187292	0.047500000000000
0.3	0.086777157401376	0.088656075000000	0.090000000000000
0.4	0.129763366662727	0.146646247985667	0.148000000000000
0.5	0.179461103563882	0.244171933030667	0.253000000000000
0.6	0.313587006669394	0.353878880000000	0.360000000000000
0.7	0.449683972599175	0.496823975095917	0.511000000000000
0.8	0.597860722776229	0.641515607222667	0.697000000000000
0.9	0.685154967050886	0.705078675000000	0.810000000000000
1.0	0.800666439370880	0.841666666666667	1.000000000000000

Table 5 presents the result of ADM and VIM numerical solution with the analytical solution for experiment 3.

Table 6. Comparison of Error for Experiment 3

a/y	Error of VIM	Error of ADM
0.1	0.000000000000000	0.000000000000000
0.2	0.000000000000000	0.000000000000000
0.3	0.000000000000004	0.000000000000000
0.4	0.000000000000174	0.000000000000100
0.5	0.000000000003191	0.000000000002448
0.6	0.000000000034435	0.000000000000363
0.7	0.000000009742310	0.000000000025112
0.8	0.000000001475060	0.000000000559862
0.9	0.000000006880829	0.000000002033417
1.0	0.000000027312667	0.000000013894145

Table 6 computes the comparison between the errors of ADM and VIM for experiment 3. The error of the ADM is lesser than that of the VIM.

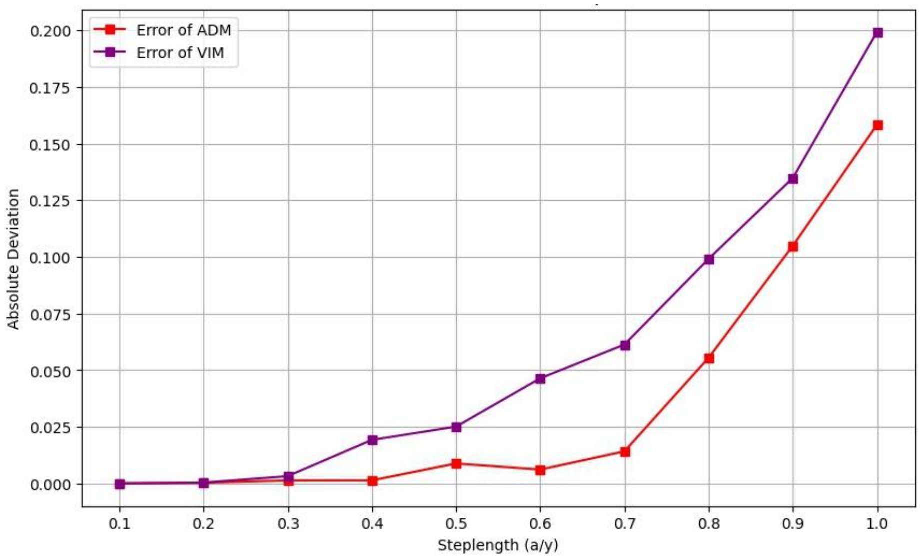

Figure 3. Error Plot for Experiment 3

Figure 3 illustrates the computed errors of ADM and VIM for experiment 3. Again, the ADM solution outperforms the VIM solution, as illustrated in the plot.

4. Conclusion

The ADM and VIM were applied to solve various linear and nonlinear Partial Differential Equations to assess these methods' effectiveness in addressing PDEs. Applying these methods resulted in some solutions, showing how the methods converge to a highly accurate exact solution. These results are thoroughly analyzed. Tables 1, 3, and 5 compute numerical solutions of ADM and VIM, tables 2, 4, and 6, and figures 1 to 3 compute the respective errors and show that the ADM method effectively solves these selected PDEs.

Numerical approximations (Tables 1, 3 and 5):

i. The tables for all three problems show the solutions from applying the two methods as well as the effectiveness of the methods in producing results that converge towards the analytical solution.
ii. A Notable difference is observed from the results of the two methods; this enhances the understanding of the accuracy of the methods and also influences selection.
iii. The results computed shows that the ADM is more effective and accurate than the VIM in solving these types of PDE problems.

Error of Numerical Approximations (Tables 2, 4 and 6; Figures 1-3):

i. The tables and graphs are representations of errors resulting from the application of the two methods.
ii. The ADM method consistently produced more accurate results and exhibited fewer errors in contrast to the VIM, thereby suggesting what method is most effective.
iii. The margin between the errors of the ADM and VIM from the figures clearly reveals that the ADM performs better.

Outcomes:

i. From the comparative analysis carried out, it is evident that the performance of the ADM is superior to that of the VIM, thereby influencing the choice of an effective numerical method to solve specific types of problems.
ii. The ADM demonstrates effectiveness and higher accuracy through its consistent convergence to the analytical solution and display of minimal error.

5. Conclusion

Various linear and nonlinear partial differential equations have been solved using ADM and VIM approaches. Both techniques yielded almost accurate results for linear and nonlinear partial differential equations, as demonstrated in problems 1 and 3, supported by the numerical data in the tables. From the analysis of Tables 1 to 6 and Figures 1 to 3, it is clear that ADM outperforms VIM in effectiveness accuracy; hence, the absolute error plots further support the conclusion. Consequently, the results have shown that ADM is a highly efficient

and accurate approach for solving linear and nonlinear partial differential equations, proving that ADM is more effective in solving linear and nonlinear PDEs than VIM. Future research will explore ADM's comparison with other numerical methods and its application to real-world problems. While this study used simplified examples to highlight accuracy and convergence, future work will incorporate practical case studies in physics, engineering, and mathematical biology to demonstrate the effectiveness of ADM and VIM.

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No conflict of interest or common interest has been declared by the authors.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Journal of Innovative Science and Engineering.

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