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Research Article

Comparison of the Results Obtained by Iman Transform with Laplace Transform

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Abstract

Many processes in the real world are characterised by principles which are defined in the form of expressions involving rates of change. Mathematically, rates are derivatives and expressions are equations so we have differential equations. Differential equations play an important role for modelling many problems in different scientific fields. Sometimes, the calculations to solve these equations can be very complex and ultimately frustrating. For this reason, many integral transform methods were proposed by researchers. However, integral transform methods can give consistent solutions to many complex problems and have many application areas such as physics, mechanics, engineering, astronomy. In this work, two integral transforms, Iman transform and the well-known Laplace transform were studied comparatively to facilitate the solution of linear ordinary differential equations with constant coefficients. Applications of these two transforms show that these integral transform methods are closely related to each other.

Keywords: Iman transform, Laplace transform, Integral transform, Differential equations.

1. Introduction

Integral transforms [1-13] such as Mohand, Ara, Kushare, Kamal, Aboodh, Mahgoub, Rishi, Emad-Falih, Rohit, Laplace, Anuj etc. have become one of the most widely used mathematical techniques to find the solutions of advanced problems of various fields such as physics, engineering, economy, astronomy etc. The most important property of these integral transforms is that they give the exact solution of a problem without longer calculations. Therefore, many scientists are interested in this field and busy in introducing new integral transforms. The oldest and most widely used integral transform is the Laplace transform, developed by P.S. Laplace (in the 1780s) when he was struggling with probability theory [13]. Other available transforms in the literature were studied by researchers at different times. For example, in the year (2022), Sanap R. et al. [15] introduced Kushare transform to solve the problems based on Newton's law of cooling. Patil D. et al. [16] stated and proved convolution theorem for Kushare transform and used it to solve convolution type Volterra integral equations of first kind. Also, Patil D. et al. [17] obtained the general solution of one-dimensional hyperbolic telegraph equation of second order by using Emad-Falih transform method.

In addition to these studies, Fadhil R.A. et al. [18] found the solution of the second kind of Linear Volterra integral equation by using HY transform without any major mathematical calculations. However, Ahmadi S.A.P. et al. [19] introduced HY transform method and applied it to solve linear ordinary Laguerre and Hermite differential equations. Maitama S. et al. [20] defined an efficient Laplace type integral transform called Shehu transform for solving both ordinary and partial differential equations. Maktoof S.F. et al. [21] proposed Emad-Sara integral transform to simplify the process of solving linear ordinary differential equations with constant coefficients in the time domain. However, N-Transform similar to Laplace and Sumudu transforms was presented by Khan in [22]. Elzaki transform was introduced by Elzaki in [23]. H-Transform was devised by Srivastava in [24].

Furthermore, a new integral transform method was suggested by Yang for solving differential equation in the steady heat-transfer problem in [25]. Sadik transform and complex Sadik transform were analyzed by Mushtt in [26]. They also made a comparison between Sadik transform and complex Sadik transform of systems of ordinary differential equations. Combined Aboodh and Reduced differential transform methods were used by Oyewumi to solve nonlinear time-dependent Fisher's type equations in [27]. Kılıcman et al. [28] explained the features of Sumudu transform and demonstrated the connections between Laplace and Sumudu transforms. Also, Elzaki and Ezaki discussed some relationships between Laplace transform and Elzaki transform in [29].

2. Definitions and Theorems

Definition 2.1 Iman transform is described for a function of exponential order in the A set as

$$A=\left\{f(t):\exists\, M,k_1,k_2>0\,,|f(t)|<M\mathbf{e}^{-\mathbf{v}^2\mathbf{t}}\right\}$$

where $f(t)$ is a dedicated function in the set A, M is a finite number and k_1, k_2 can be finite or infinite.
Iman transform denoted by operator I is defined as [30]

$$I\{f(t)\}=R(v)=\frac{1}{v^2}\int_0^\infty e^{-v^2t}f(t)\,dt,\quad t\geq 0,\ \ k_1\leq v\leq k_2. \tag{2.1}$$

Definition 2.2 If $\boldsymbol{R(v)}$ is called the Iman transform of $\boldsymbol{f(t)}$, then $\boldsymbol{f(t)}$ is called the inverse Iman transform of $\boldsymbol{R(v)}$,and it can be expressed as

$$\boldsymbol{f(t) = I^{-1}\{R(v)\}}. \tag{2.2}$$

Definition 2.3 Laplace transform of the function $f(t)$ denoted by operator L is defined as [1, 10, 14, 29]

$$L\{f(t)\} = K(s) = \int_0^\infty e^{-st} f(t)\, dt, \quad t \geq 0, \ \ Re(s) > 0. \tag{2.3}$$

Definition 2.4 If $\boldsymbol{K(s)}$ is called the Laplace transform of $\boldsymbol{f(t)}$, then $\boldsymbol{f(t)}$ is called the inverse Laplace transform of $\boldsymbol{K(s)}$, and it can be expressed as

$$\boldsymbol{f(t) = L^{-1}\{K(s)\}}. \tag{2.4}$$

Theorem 2.1 Let $I\{f(t)\} = R(v)$. Iman transform of first and second derivatives of $f(t)$ are given as [30, 31]

- $I\{f'(t)\} = v^2 R(v) - \frac{1}{v^2} f(0),$ (2.5)
- $I\{f''(t)\} = v^4 R(v) - f(0) - \frac{1}{v^2} f'(0).$ (2.6)

Theorem 2.2 Let $L\{f(t)\} = K(s)$. Laplace transform of first and second derivatives of $f(t)$ are given as [1, 10, 14, 29]

- $L\{f'(t)\} = sK(s) \ - f(0),$ (2.7)
- $L\{f''(t)\} = s^2 K(s) \ - sf(0) - f'(0).$ (2.8)

In the table below, Iman transform of some elementary functions [30] were given.

Table 1: Iman transform of some elementary functions

Sequence	**Function $(\boldsymbol{f(t)})$**	**Iman transform $\boldsymbol{I\{f(t)\}}$**
1	1	$\frac{1}{v^4}$
2	t^n	$\frac{n!}{v^{2n+4}}$
3	e^{at}	$\frac{1}{v^4 - av^2}$
4	$sinat$	$\frac{a}{v^2(v^4 + a^2)}$
5	$cosat$	$\frac{1}{v^4 + a^2}$
6	$sinhat$	$\frac{a}{v^2(v^4 - a^2)}$
7	$coshat$	$\frac{1}{v^4 - a^2}$

And now in the table below, Laplace transform of some elementary functions [10, 14] were given.

Table 2: Laplace transform of some elementary functions

Sequence	Function $(f(t))$	Laplace transform $L\{f(t)\}$
1	1	$\frac{1}{s}$
2	t^n	$\frac{n!}{s^{n+1}}$
3	e^{at}	$\frac{1}{s-a}$
4	$sinat$	$\frac{a}{(s^2+a^2)}$
5	$cosat$	$\frac{s}{s^2+a^2}$
6	$sinhat$	$\frac{a}{(s^2-a^2)}$
7	$coshat$	$\frac{s}{s^2-a^2}$

Theorem 2.3 Let $R_1(v)$, $R_2(v)$, $R_3(v)$, $\cdots$, $R_n(v)$ be the Iman transforms of the functions $f_1(t)$, $f_2(t)$, $f_3(t)$, $\cdots$, $f_n(t)$ respectively, then

$$I\{xf_1(t) + yf_2(t) + zf_3(t) + \cdots + tf_n(t)\} = xR_1(v) + yR_2(v) + zR_3(v) + \cdots + tR_n(v) \tag{2.9}$$

where $x, y, z, \cdots, t$ are arbitrary constants.

Theorem 2.4 Let $K_1(s)$, $K_2(s)$, $K_3(s)$, $\cdots$, $K_n(s)$ be the Laplace transforms of the functions $f_1(t)$, $f_2(t)$, $f_3(t)$, $\cdots$, $f_n(t)$ respectively, then

$$L\{xf_1(t) + yf_2(t) + zf_3(t) + \cdots + tf_n(t\} = xK_1(s) + yK_2(s) + zK_3(s) + \cdots + tK_n(s) \tag{2.10}$$

where $x, y, z, \cdots, t$ are arbitrary constants.

3. Applications of Linear Ordinary Differential Equations

Consider that the first-order linear ordinary differential equation with the initial condition $y(0) = a$ is given as

$$\frac{dy}{dt} + ky = g(t) \quad , \quad t > 0 \tag{3.1}$$

where Iman transform of $g(t)$ as a function of "t" is denoted by $G(v)$, Laplace transform of $g(t)$ is denoted by $G(s)$, and a, k are constants.

Applying Iman transform on both sides of (3.1) and substituting the initial condition, we have

$$I\left\{\frac{dy}{dt}\right\} + kI(y) = I\{g(t)\},$$

$$v^2 I(y) - \frac{1}{v^2} y(0) + kI(y) = G(v),$$

$$v^2 I(y) + kI(y) = G(v) + \frac{a}{v^2},$$

$$I(y) = \frac{G(v)}{(v^2+k)} + \frac{a}{v^2(v^2+k)}. \tag{3.2}$$

Therefore, we can find the solution by applying inverse Iman transform of equation (3.2).

Now if we apply Laplace transform on both sides of (3.1), we have

$$L\left\{\frac{dy}{dt}\right\} + kL(y) = L\{g(t)\},$$

$$sL(y) - y(0) + kL(y) = G(s),$$

$$sL(y) + kL(y) = G(s) + a,$$

$$L(y) = \frac{G(s)}{s+k} + \frac{a}{s+k}. \qquad (3.3)$$

Then we find the solution by applying inverse Laplace transform of equation (3.3) in the step above. Consider that the second-order linear ordinary differential equation with the initial conditions $y(0) = a$ and $y'(0) = b$ is given as

$$\frac{d^2y}{dt^2} + k\frac{dy}{dt} + ly = g(t) \quad , \quad t > 0 \qquad (3.4)$$

where Iman transform of $g(t)$ as a function of "t" is denoted by $G(v)$, Laplace transform of $g(t)$ as a function of "t" is denoted by $G(s)$, and a, b, k, l are constants.

Applying Iman transform on both sides of (3.4) and substituting the initial conditions, we obtain

$$I\left\{\frac{d^2y}{dt^2}\right\} + kI\left\{\frac{dy}{dt}\right\} + lI(y) = I\{g(t)\},$$

$$\left\{v^4I(y) - y(0) - \frac{1}{v^2}y'(0)\right\} + k\left\{v^2I(y) - \frac{1}{v^2}y(0)\right\} + lI(y) = G(v),$$

$$I(y)(v^4 + kv^2 + l) = G(v) + a + \frac{1}{v^2}(b + ak),$$

$$I(y) = \frac{G(v)}{(v^4+kv^2+l)} + \frac{a}{(v^4+kv^2+l)} + \frac{(b+ak)}{v^2(v^4+kv^2+l)}. \qquad (3.5)$$

Hence, we find the solution by applying inverse Iman transform in the step above. Now if we apply Laplace transform on both sides of (3.4), we obtain

$$L\left\{\frac{d^2y}{dt^2}\right\} + kL\left\{\frac{dy}{dt}\right\} + lL(y) = L\{g(t)\},$$

$$\{s^2L(y) - sy(0) - y'(0)\} + k\{sL(y) - y(0)\} + lL(y) = G(s),$$

$$L(y)(s^2 + ks + l) = G(s) + b + a(s + k),$$

$$L(y) = \frac{G(s)}{(s^2+ks+l)} + \frac{b}{(s^2+ks+l)} + \frac{a(s+k)}{(s^2+ks+l)}. \qquad (3.6)$$

Then we find the solution by applying inverse Laplace transform in the step above.

Example 3.1 Assume that the first-order differential equation is given as

$$y' + 3y = cost \ , \ y(0) = 2 \qquad (3.7)$$

Applying Iman transform on both sides of (3.7), we get

$$I\{y'\} + 3I(y) = I\{cost\},$$

$$v^2I(y) - \frac{1}{v^2}y(0) + 3I(y) = \frac{1}{v^4+1},$$

$$I(y) = \frac{2v^4+v^2+2}{v^2(v^4+1)(v^2+3)}.$$

The inverse Iman transform of the equation above provides the solution

$$y(t) = \frac{3}{10}\, cost + \frac{1}{10}\, sint + \frac{17}{10}\, e^{-3t}. \tag{3.8}$$

Now if we apply Laplace transform on both sides of (3.7), we get

$$L\{y'\} + 3L(y) = L\{cost\},$$
$$sL(y) - y(0) + 3L(y) = \frac{s}{s^2+1},$$
$$L(y) = \frac{2s^2+s+2}{(s^2+1)(s+3)}.$$

Applying inverse Laplace transform in the step above, we write

$$y(t) = \frac{17}{10}\, e^{-3t} + \frac{3}{10}\, cost + \frac{1}{10}\, sint. \tag{3.9}$$

Example 3.2 Consider the following equation is given as

$$y' + y = sint \ , \ \ y(0) = 1 \tag{3.10}$$

Applying Iman transform on both sides of (3.10), we get

$$I\{y'\} + I(y) = I\{sint\},$$
$$v^2 I(y) - \frac{1}{v^2} y(0) + I(y) = \frac{1}{v^2(v^4+1)},$$
$$I(y) = \frac{v^4+2}{v^2(v^4+1)(v^2+1)}.$$

The inverse Iman transform provides the solution as

$$y(t) = -\frac{1}{2} cost + \frac{1}{2}\, sint + \frac{3}{2}\, e^{-t}. \tag{3.11}$$

Now if we apply Laplace transform on both sides of (3.10), we have

$$L\{y'\} + L(y) = L\{sint\},$$
$$sL(y) - y(0) + L(y) = \frac{1}{s^2+1},$$
$$L(y) = \frac{s^2+2}{(s^2+1)(s+1)}.$$

Applying inverse Laplace transform in the step above, we write

$$y(t) = -\frac{1}{2} cost + \frac{1}{2}\, sint + \frac{3}{2}\, e^{-t}. \tag{3.12}$$

Example 3.3 Assume that the following equation is given as

$$y'' - 3y' + 2y = 0 \ , \ \ y(0) = 1 \, , \ \ y'(0) = 4 \tag{3.13}$$

Taking Iman transform for both sides of (3.13), we have

$$I\{y''\} - 3I\{y'\} + 2I(y) = 0,$$
$$v^4 I(y) - y(0) - \frac{1}{v^2} y'(0) - 3v^2 I(y) + \frac{3}{v^2} y(0) + 2I(y) = 0,$$
$$(v^4 - 3v^2 + 2) I(y) = 1 + \frac{1}{v^2}\, ,$$

$$I(y) = \frac{v^2+1}{v^2(v^2-2)(v^2-1)}.$$

After simple calculations, we get fractional parts

$$I(y) = \frac{3}{v^2(v^2-2)} - \frac{2}{v^2(v^2-1)}.$$

The inverse Iman transform of the equation above will be

$$y(t) = 3e^{2t} - 2\,e^{t}. \tag{3.14}$$

Now if we apply Laplace transform on both sides of (3.13), we obtain

$$L\{y''\} - 3L\{y'\} + 2L(y) = 0,$$
$$s^2L(y) - sy(0) - y'(0) - 3\{sL(y) - y(0)\} + 2L(y) = 0,$$
$$L(y) = \frac{s+1}{(s-2)(s-1)}.$$

Applying inverse Laplace transform in the step above, we find

$$y(t) = 3e^{2t} - 2\,e^{t}. \tag{3.15}$$

Example 3.4 Assume that the following equation,

$$y'' - 3y' + 2y = 4e^{3x}, \qquad y(0) = -3, \quad y'(0) = 5 \tag{3.16}$$

Applying Iman transform on both sides of (3.16), we have

$$I\{y''\} - 3I\{y'\} + 2I(y) = I\{4e^{3x}\},$$
$$v^4I(y) - y(0) - \frac{1}{v^2}y'(0) - 3v^2I(y) + \frac{3}{v^2}y(0) + 2I(y) = \frac{14}{v^2} - 3 + \frac{4}{v^4-3v^2},$$
$$(v^4 - 3v^2 + 2)I(y) = \frac{14}{v^2} - 3 + \frac{4}{v^4-3v^2},$$
$$I(y) = \frac{23v^2-3v^4-38}{v^2(v^2-3)(v^2-2)(v^2-1)}.$$

After simple calculations, we get

$$I(y) = \frac{2}{v^2(v^2-3)} + \frac{4}{v^2(v^2-2)} - \frac{9}{v^2(v^2-1)}.$$

The inverse Iman transform of the equation above provides the solution

$$y(t) = 2e^{3t} + 4e^{2t} - 9e^{t}. \tag{3.17}$$

Now if we apply Laplace transform on both sides of (3.16), we have

$$L\{y''\} - 3L\{y'\} + 2L(y) = 0,$$
$$s^2L(y) - sy(0) - y'(0) - 3\{sL(y) - y(0)\} + 2L(y) = \frac{4}{s-3},$$
$$L(y) = \frac{-3s^2+23s-38}{(s-3)(s^2-3s+2)}.$$

Finally, applying inverse Laplace transform in the step above, we find

$$y(t) = 2e^{3t} + 4e^{2t} - 9e^{t}. \tag{3.18}$$

4. Conclusions

We have seen that the Iman transform method proposed in this study and compared with the Laplace transform, is accurate and efficient as an alternative approach to solve linear ordinary differential equations without longer calculations. The examples in application section reveal that both methods are closely connected to each other. However, the examples here will be solved with another integral transform methods available in the literature. Since Iman transform is a new integral transform method, we believe that the number of studies in different fields will increase day by day.

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