

**Original Article** 

# The Best Fit Flood Probability Distribution for Alibeyköy Basin in İstanbul, Türkiye

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## Abstract

Determination of peak flow rates is crucial in reducing the economic and social impact of floods. Therefore, the investigation of various methods for estimating floods is of paramount importance. Flood frequency analysis can be used as a practical method in predicting the peak flow values as the flood peaks have return periods that are typically much larger than the recording length. In this study, seven different probability distributions (normal (N), two-parameter lognormal (LN2), three-parameter lognormal (LN3), extreme value type I (Gumbel), generalized extreme value (GEV), Pearson Type III (P3) and Log-Pearson Type III (LP3)) are employed for flood frequency analysis of Alibeyköy Watershed using 44-years of measured annual maximum flow. K-S and PPCC tests are applied to determine the most suitable distributions to estimate the flood flow rate. Based on these tests, GEV and Gumbel distributions appear to be the most preferable distributions in flood flow estimation.

Keywords: flood frequency analysis, probability distribution, return period, peak flow

## 1. INTRODUCTION

Flood frequency analysis is employed for the best fitting of a probability distribution to observed data to make predictions about the occurrence of floods. Long data sets are needed to accurately predict of flood return periods. However, most of the time, water resources engineers suffer from a lack of data and thus, probabilistic approaches are used in flood predictions. The selection of an appropriate fit among many existing probability distributions is the most important stage of such studies, which is referred as flood frequency analysis. Flood frequency analysis deals, in fact, not only with maximum flow rates but also with minimum flow rates. Önöz and Bulu [1] employed low flow frequency analysis to determine the minimum downstream release requirements from hydropower, water supply, cooling plants, and other facilities. Many studies in the literature point out the importance of flood frequency analysis in water resources management [2, 3, 4]. Flood frequency analyses are carried out in a large extent both in Turkey [5] and in the world [6,7,8]. L-moments have been extensively used as a tool in regional flood frequency analyses [9-15]. While GEV, Gumbel, Normal, two parameters Log-Normal, three parameters Log-Normal, Gamma, Pearson Type III, and Log-Pearson Type III distributions appear as selected probability curves in flood peak flow fitting [16-17], GEV distribution is commonly found as the most suitable distribution function among the studies conducted on flood frequency analyses [18-20]. [21] point out several factors which are effective

on the reliance of statistical flood frequency analysis such as the selected probability distribution function, estimation of the function parameters, possible outliers, and length of the observed flood series. [22] point out the importance of estimating the T-year flood discharge, which is the discharge once exceeded on the average in a period of T years, as the ultimate interest of flood frequency analysis. The objective of this study is to determine the best fit probability distributions for estimating T-year flood recurrence intervals of the rivers in Alibeyköy basin in İstanbul, Turkey. For this purpose, seven probability distributions called the normal (N), two-parameter lognormal (LN2), three-parameter lognormal (LN3), extreme value type I (Gumbel), generalized extreme value (GEV), Pearson Type III (P3) and Log-Pearson Type III (LP3), are considered. The statistical analyses are conducted using the yearly maximum flow rate data recorded for 44 years on Alibeyköy Pirinçci Stream to find the peak flow rates with different return periods. L- Moment methods are employed in these analyses. Finally, Kolmogorov-Smirnov (K-S) statistical tests and probability plot correlation coefficient (PPCC) are performed to select the best-fit flood probability distribution for Alibeyköy Watershed.

## 2. METHODOLOGY

## **2.1 Probability Distribution Functions**

Many flood frequency models have been suggested in the literature, but none of them has been accepted universally [23]. In order to achieve some degree of uniformity in the determining of flood quantiles, some countries have agreed to adopt a certain distribution function. For example, the Log Pearson Type III was recommended by the US Water Resources Council (USWRC) in 1967 for use in the USA and the general extreme value distribution (GEV) was suggested by the Institute of Hydrology, UK, for use in the UK and Ireland. In this study, seven probability distributions are considered to predict flood discharges in Alibeyköy Watershed, Turkey. These distributions are normal (N), two parameters lognormal (LN 2), three parameters lognormal (LN 3), Extreme Value Type I (Gumbel), Generalized Extreme Value (GEV), Pearson Type III (P3) and Log-Pearson Type III (LP3), which are widely used in hydrologic frequency analysis. These distributions and their probability functions (PDF) are presented in Table 1 [24]. In these equations, x is the observed value. Further details of these distributions can be found in a book by Rao and Hamed [26].

Distribution type	<b>Probability density function</b>	Parameters
		$\mu$ =mean (location
Normal (N)	$1 \frac{1}{(2 - 2)^2}$	parameter)
	$f(x) = \frac{1}{\sqrt{2\sigma^2}} \exp^{2\sigma^2(x-\mu)^2}$	$\sigma$ =Standard
	$\sigma_{\sqrt{2\pi}}$	deviation (scale
		parameter)
	<b>F 1 1 1</b>	$\mu_y = \text{location}$
Two-Parameter	$f(x) = \begin{bmatrix} 1 \\ - \end{bmatrix} \begin{bmatrix} (\ln x - \mu_y)^2 \end{bmatrix}$	parameter
lognormal (LN2)	$\int (x) - \frac{1}{x \cdot \sigma_y \sqrt{2\pi}} \cdot \exp \left[ - \frac{1}{2\sigma_y^2} \right]$	$\sigma_{\rm y} = {\rm scale}$
		parameter
		$\mu_{\rm y} = {\rm location}$
Three-Parameter Lognormal (LN3)		parameter
	$f(x) = 1 \left[ \log(x-a) - \mu_y \right]^2$	$\sigma_{\rm y} = {\rm scale}$
	$\int (\chi) - \frac{1}{(x-a)\sigma_y\sqrt{2\pi}} \cdot \exp\left[-\frac{1}{2\sigma_y^2}\right]$	parameter
		a = shape
		parameter

Table 1. Statistical distribution and functions [24, 25]

Generalized Extreme		$\sigma$ =scale parameter
Value (GEV)	$f(x) = \frac{1}{k} \exp(-(1+kz)^{-\frac{1}{k}})(1+kz)^{-1-\frac{1}{k}}$	$(\sigma > 0)$
$(\mathbf{x} - \mathbf{u})$	$\int (x)^2 = \exp(-(1+x^2)) + (1+x^2)$	K – Sliape
$1+k\frac{(x-\mu)}{\sigma}$ for $k\neq 0$	$f(x) = \frac{1}{\sigma} \exp(-z - \exp((-z))$	z-location
-∞ <x<∞ for="" k="0&lt;/td"><td></td><td>parameter</td></x<∞>		parameter
The extreme value type I		
(Gumbel)	$x \to 1  [x - \xi  x - \xi_{\lambda}]$	$\zeta$ =location
	$f(x) = \frac{1}{\alpha} exp\left[-\frac{1}{\alpha} - exp\left(-\frac{1}{\alpha}\right)\right]$	parameter
-∞ <x<∞< td=""><td></td><td><math>\alpha</math>=scale parameter</td></x<∞<>		$\alpha$ =scale parameter
		α=shape
	$f(x) = \frac{\beta}{\Gamma(\alpha)} (\beta(x-\xi))^{\alpha-1} \exp\left[-(\beta)x - \xi\right]$	parameter
The Deerson Tune III		$(\alpha > 0)$
(P2)		$\beta$ =scale parameter
(F3)		$(\beta \# 0)$
		ξ=location
		parameter
		α=shape
		parameter
Log-Pearson Type III (LP3)	$f(x) = \frac{ \beta }{(\beta)} (\beta) (\ln(x) - \xi)^{\alpha-1} \exp\left[-(\beta) \ln(x) - \xi\right]$	$(\alpha > 0)$
	$\int (x) - \alpha \Gamma(\alpha) (p(\Pi(x) - \zeta)) = \exp[-(p) \Pi(x)]$	$\beta$ =scale parameter
	ξ]	$(\beta \# 0),$
		ξ=Location
		parameter

## **2.2 Parameter estimation**

After selecting the probability distribution functions, the next step is the estimation of the location, scale, and shape parameters. The estimated parameters are then used in the probability distribution functions to calculate quantile estimates for different return periods or to calculate the return period for a given flood magnitude. There are many methods for parameter estimation, such as the method of moments, the probability-weighted moments method, the maximum likelihood method, the least squares method, maximum entropy, mixed moments, the generalized method of the moments, and the incomplete means method. The details of these methods are already available in the literature. Among these methods, statistical moments, L-moments, and maximum probability methods are used frequently in the determination of the relevant parameters of probability distribution functions [27]. In this study, L-moments method is employed in parameter estimation.

#### 2.3 L-moments

In the last century, one of the most significant scientific contributions to statistical hydrology was made by Hosking [28] with the L-moments, which are special cases of probability-weighted moments. The advantages of the L-moments can be summarized as follows: (i) they characterize a wider range of distributions than conventional moments, (ii) they are less sensitive to outliers in the data, (iii) they approximate their asymptotic normal distribution more closely, and (iv) they are nearly unbiased for all combinations of sample sizes and populations [29]. L-moments are alternatives to determine the main characteristics of the probability distribution of hydrological data. L-moments may be considered a linear combination of data series in ascending order. The general expression for probability-weighted moments is given as follows [25]:

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$$b_{r} = \frac{1}{N} \sum_{j=1}^{N-r} \frac{\binom{N-j}{r} x_{j}}{\binom{N-1}{r}}$$
(1)

According to this formula:

$$b_0 = \mu \tag{2}$$

$$b_1 = \sum_{j=1}^{N-1} \frac{(N-j)x_j}{N(N-1)}$$
(3)

$$b_2 = \sum_{j=1}^{N-2} \frac{(N-j)(N-j-1)x_j}{N(N-1)(N-2)}$$
(4)

$$b_{3} = \sum_{j=1}^{N-3} \frac{(N-j)(N-j-1)(N-j-2)x_{j}}{N(N-1)(N-2)(N-3)}$$
(5)

The usage of the confident estimations of probability-weighted moments in the charts of L-moments and regional analyses is suggested. L-moments can be calculated by using probability-weighted moments as follows:

$$\lambda_2 = 2b_1 - b_0 \tag{7}$$

$$\lambda_3 = 6b_2 - 6b_1 + b_0 \tag{8}$$

$$\lambda_4 = 20b_3 - 30b_2 + 12b_1 - b_0 \tag{9}$$

L-skewness 
$$(\tau 3) = \lambda_3 / \lambda_2$$

L-kurtosis(
$$\tau 4$$
) = $\lambda_4/\lambda_2$  (10)  
L-CV ( $\tau$ ) =  $\lambda_2/\lambda_1$ 

$$E \in V(0)$$
  $M_2/M_1$ 

## 2.4 Kolmogorov-Smirnov (K-S) test

Two goodness of-fit-tests are used for evaluating the suitability of different probability distributions in this study. A procedure based on the expected number of exceedances of a certain flood event was used.

In Kolmogorov-Smirnov (K-S) test, the observed data series is sorted in ascending order first. Then, for each observed value of xi, its probability of non-exceedance F(x) is an empirical distribution function calculated using a plotting position formula. S(x) is the theoretical cumulative distribution of the tested distribution computed using the chosen probability distribution. According to the K-S test, the largest value of the differences between these two probabilities is considered for the goodness-of-fit test criterion.

From K-S' table, according to the acceptable level of significance  $\alpha$ , commonly taken as 90% or 95%, and the number of elements in the sample series, n, Dtable is obtained. If Dtable  $\geq$ Dmax, then the chosen probability distribution is said to fit to the observed sample series [5].

$$F(x) = P(X \le x) \tag{11}$$

$$D=\max \left[F(x)-S(x)\right] \tag{12}$$

## 2.5 Probability plot correlation coefficient (PPCC) test

The adequacy of a fitted distribution can be evaluated by the PPCC coefficient, which is essentially a measure of the linearity of the probability plot [30]. PPCC is a powerful goodness-of-fit test for normality developed by Filliben and Looney, and Gulledge [31]. This test is readily extendible for testing some non-normal distributional hypotheses. Filliben's PPCC test statistic is defined as the product moment correlation coefficient (r) between the ordered observations yi and the corresponding fitted quantiles Qi which is determined by the plotting position formula for each yi. The test statistic is defined by:

$$r = \frac{\sum_{i=1}^{N} (Y_i - \bar{Y})(Q_i - \bar{Q})}{\sqrt{\sum_{i=1}^{N} (Y_1 - \bar{Y})^2 (Q_i - \bar{Q})}}$$
(13)

Y and Q represent the mean values of the observation Yi and the fitted quantile Qi, respectively, and N is the sample size [15]. This test is seen to be much more powerful than other tests. In many cases, the two-parameter distributions are rejected.

## 2.6 Study Area: Alibeyköy Watershed

Alibeyköy Watershed is located on the European Continental side of Istanbul in Turkey. It has a drainage area of 161 km2 and supplies an important portion of Istanbul's drinking water. There are 10 streams that gather overland flow generated over the basin. These streams are Cebeci stream, Boğazköy stream, Bolluca stream, Kocaman stream, Çıplak stream, Ayvalı stream, Elmalı stream, Gülgen stream, Malkoç stream, Çiftepınar stream. The land morphology of the great part of the watershed is in the form of sandy clay loam. The altitude of the watershed is between 30-170 m in the topographic boundaries. Alibeyköy Watershed is composed of 23% of agricultural and pastureland, 15% of residential and industrial areas, 60% of forest, and 2% of dam area. However, there is a great potential for population growth due to the new developments of infrastructures in the basin. These new infrastructures include third Bosphorus Bridge, third Istanbul airport and Canal Istanbul project. Therefore, high urbanization is expected in the next 10 years and these changes will have negative effects on the ecosystem in the future if no action is taken. Moreover, settlements on this watershed especially near the mainstream are under flood risk. Selection of Alibeyköy Watershed as the study site is thus crucial. The boundary of Alibeyköy Watershed and Alibeyköy Dam are shown in Figure 1.



Figure 1. General view of Alibeyköy Watershed and location of rain gauge and flow meter

In this study, annual maximum flow data recorded for 44 years at Alibeyköy gauge station located on Pirinççi Stream (DSİ\_AGİ\_D02A047) is used. The flow meter is located downstream of the Pirinççi Stream (Figure 2).



Figure 2. General view of flow meter in Alibeyköy Watershed

The annual maximum flow data for 1965-2020 were obtained from State Hydraulic Works of Turkey. The graphics of the annual maximum data series is given in Figure 3.



Figure 3. Annual maximum peak flow rates of Pirincci stream in Alibeyköy Basin

# 3. RESULTS AND DISCUSSION

The minimum and maximum values of the annual maximum flow data series for the D02A047 gauge station are provided in Table 2. In addition, the statistics using classical moments are calculated using the annual maximum flow data and given in Table 2.

Parameters	Values
Number of data ( <i>N</i> )	44
Minimum flow (m <sup>3</sup> /s)	6.16
Maximum flow (m <sup>3</sup> /s)	165
Mean flow $(m^3/s)$	56.12
Standard deviation (m <sup>3</sup> /s)	34.47
Coefficient of skewness	1.05
Coefficient of quartile skewness	0.33
Median (m <sup>3</sup> /s)	44.30
Destination between quartiles	44.95
Median absolute deviation (m <sup>3</sup> /s)	11.82
Variance	1187.86

	Table 2. Minimum an	d maximum	values	of flow	data and	the corres	ponding	statistics
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Probability-weighted moments and L-moments are computed for D02A047 gauge station data and the relevant parameters are presented in Table 3. The statistical parameters are location, scale, and shape parameters. Using probability-weighted moments (PWMs), L-moments are computed. Then, L-moment ratios are defined, which are L-coefficient of variation, L-skewness, and L-kurtosis.

D02A047 gauge	L-Moments and rates
$b_0$	56.12
$b_1$	38.82
<b>b</b> <sub>2</sub>	30.29
<b>b</b> <sub>3</sub>	25.11
(Location) L <sub>1</sub>	56.12
(Scale) L <sub>2</sub>	21.52
(Shape) L <sub>3</sub>	4.96
(threshold) L <sub>4</sub>	3.06
(Variation) LC <sub>v</sub>	0.38
(Skewness) LCs	0.09
L-kurtosis	0.14

Table 3. Probability weighted moments and L-moments

The parameters of the N distribution are the mean  $\mu$  and standard deviation  $\sigma x$ . Non-normal distributed variables can be adjusted to the normal distribution by means of a suitable distribution. One of these transformation methods is computing the logarithms (y=lnx). In this case, logarithmic mean  $\mu$  and standard deviation  $\sigma y$  will be the parameters of the LN2 distribution. For the Gumbel distribution, scale and location parameters were estimated by PWMs and L-moments. The GEV, P3, LN3, and LP3 distributions contain a shape parameter in addition to location and scale parameters, which are also estimated by PWM sand L-moments.

Just as the log-normal distribution, which represents the logarithm of the normal distribution of the variable x, LP3 represents the logarithm of the P3. Moreover, the 3-parameter log-normal distribution (LN3) represents the logarithm of the normal distribution with an additional parameter xo corresponding to a lower boundary. The estimated parameters of 2 and 3-parameter distributions are presented in Table 4 and Table 5.

Then, the maximum flow rate for return periods of 50, 100, 200, and 500 years are predicted using seven different probability distribution functions and are given in Table 6. As it can be seen from this Table, for low return periods (i.e. 50 and 100 years), LN2 predicts the flood flow the lowest, whereas P3 predicts the flood flow the highest. For high return periods (i.e. 200 and 500 years), N predicts the lowest flood flow, whereas LP3 predicts the highest flood flow. In general, the flood flow results for LP3 are significantly higher than the results of the rest of the probability distribution functions as the return period increases. The maximum flow data is measured as 165 m<sup>3</sup>/s at a gauging station in Alibeyköy Watershed for 44 years of annual maximum flow data. As it can be seen in Table 6 that this value is between the values calculated by Gumbel and GEV distribution functions for 50-year return period.

The comparison of 2-parameter distributions with the observed data and 3-parameter distributions with the observed data are presented in Figures 4 and 5, respectively. As it can be seen from Figure 4 that Gumbel fits the observed data better than N and LN2. N represents the flood flow for low return periods better than the ones for high return periods. When 3-parameter distributions are compared, GEV fits the observed data better than LN3, P3, and LP3.

Table 4. Parameter estimation of 2-parameter distribution	utions
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N		L	N2	Gun	Gumbel	
μ	$\sigma_x$	μ	$\sigma_y$	ζ	α	
56.12	34.47	3.38	0.57	31.06	38.19	

Table 5. Parameter estimation of 3-parameter distributions

	LN3			GEV			F	23			L	.P3	
	σ	ra	k	a	11		K (	$C_{\rm sx}$ )			K	$(C_{\rm sy})$	
$\mu_x$	$o_x$	$\lambda_0$	ĸ	u	и	50	100	200	500	50	100	200	500
3.25	0.92	-15.98	-0.09	28.29	36.96	2.58	3.09	4.74	5.23	2.58	3.09	4.74	5.23



Figure 4. 2-parameters probability distributions and observed data in the Pirinçci stream.



Figure 5. 3-parameters probability distributions and observed data in the Pirincci stream.

Finally, the goodness of-fit-test for each probability distribution function is performed using K-S and PPCC methods. The results of K-S and PPCC methods are presented in Table 7 and Table 8, respectively. Based on K-S test, all distributions used in this study are found as suitable. Based on the PPCC test, among 2parameter distributions, only Gumbel is found the suitable distribution and among 3-parameter distributions, only GEV is found as the suitable distribution. Thus, GEV and Gumbel distributions, which perform the best results for the goodness of fit tests, are chosen among the probability distributions investigated in this study for decisions in Alibeyköy watershed planning and management.

Station no D02A047						
	Flood flow (m <sup>3</sup> /s) for various Retur periods (year)					
Distribution type	50	100	200	500		
Normal	135	145	155	166		
Log-Normal2	112	133	155	185		
Log-Normal3	148	164	179	196		
Gumbel	159	181	203	231		
GEV	170	199	230	274		
Pearson Type III	179	208	305	334		
Log-Pearson Type III	155	211	585	791		

Table 6. Various return periods of flood flow estimated for

Table 7. Results of K-S Test

Distribution	Critical	alues ®	Suitableness of data*
	Calculated	Critical	
Normal	0.0141	0.20503	Compatible
Log-Normal 2	0.0080	0.20503	Compatible

Log-Normal 3	0.0222	0.20503	Compatible
Pearson III	0.0222	0.20503	Compatible
Log-Pearson III	0.0222	0.20503	Compatible
GEV	0.0136	0.20503	Compatible
Gumbel	0.0127	0.20503	Compatible

\*If calculated r is smaller than critical r, it is suitable.

Distribution	Critical values ®		Suitableness of data*
	Calculated	Critical	
Normal	0.955	0.977	Not compatible
Log-Normal 2	0.902	0.977	Not compatible
Log-Normal 3	0.904	0.977	Not compatible
Pearson III	0.896	0.940	Not compatible
Log-Pearson III	0.822	0.940	Not compatible
GEV	0.985	0.977	Compatible
Gumbel	0.988	0.970	Compatible

#### Table 8. Results of PPCC Test

\*If calculated r is bigger than critical r, it is suitable.

#### 4. CONCLUSION

Flood flow rate is an important hydrologic parameter in determining flood risk, managing water resources, and designing hydraulic structures such as dams, spillways, culverts, and irrigation ditches. The estimate of the design event should be fairly accurate to avoid excessive costs in case of overestimation of the flood magnitude or excessive damage and even loss of human lives in case of underestimation of the flood potential. This paper presents a case study for prediction of peak flow rates with different return periods for Alibeyköy Watershed. Several probability distribution functions are fitted using annual maximum flow data measured on Pirinççi Stream, and K-S and PPCC tests are employed to determine their performance. Based on the analyses carried out, the following conclusions are drawn from this study:

1. Among the commonly used distributions in hydrology (N, LN2, LN3, GEV, Gumbel, P3, LP3), the GEV and the Gumbel distributions are found as the most suitable candidates in representing the annual maximum flows of rivers of Alibeyköy basin. Therefore, the other distributions were suggested as secondary methods to estimate these quantiles.

2. The estimated flood values can be used in hydraulics, hydrology, and engineering studies related to the design and operation of hydraulic structures (bridges, culverts, dams, erosion-control structures), especially in urbanized areas so that decision makers can accurately plan watershed management strategies and protect water resources and ecology.

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