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Research Article / Araştırma Makalesi

Fractional time derivative on fluid flow through horizontal microchannel filled with porous material / Gözenekli malzeme ile doldurulmuş yatay mikrokanaldan akışkan akışında kesirli zaman türevi

Muhammad Lawan Kaurangini[1], Isyaku Shu'aibu Abdulmumini[2], Umar Muhammad Abubakar[3]*

[1] *Department of Mathematics, Faculty of Computing and Mathematical Sciences, Aliko Dangote University of Science and Technology, Wudil, Kano State, Nigeria*
[2] *Department of Mathematics and Statistics, Yobe State University, Damaturu, Nigeria*
[3] *Department of Mathematics, Faculty of Computing and Mathematical Sciences, Aliko Dangote University of Science and Technology, Wudil, Kano State, Nigeria*

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ABSTRACT

Fractional time derivative is considered in the description of the unsteady fluid flow through a horizontal microchannel filled with porous material. The resultant governing equations were solved using the Laplace transform technique and the method of undetermined coefficient in the Laplace domain. The Riemann-sum approximation approach was then utilized to obtain the solution in the time domain. The results were then simulated and presented as line graphs utilizing MATLAB (R2015b) to study the effects of the parameters involved in the fluid flow.

ÖZET

Kesirli zaman türevi, gözenekli malzeme ile doldurulmuş yatay bir mikrokanaldan geçen kararsız akışkan akışının tanımlanmasında dikkate alınmıştır. Elde edilen yönetici denklemler Laplace dönüşümü tekniği ve Laplace alanında belirlenmemiş katsayı yöntemi kullanılarak çözülmüştür. Riemann-toplam yaklaşımı daha sonra zaman alanında çözümü elde etmek için kullanılmıştır. Sonuçlar daha sonra simüle edilmiş ve akışkan akışına dahil olan parametrelerin etkilerini incelemek için MATLAB (R2015b) kullanılarak çizgi grafikler halinde sunulmuştur.

* Corresponding author, e-mail: umabubakar347@gmail.com

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Nomenclature

Da	Darcy number	$\boldsymbol{y'}$	dimensional y-coordinate
$\boldsymbol{\rho}$	fluid density	$\boldsymbol{u_f}$	dimensionless velocity in the clear fluid region
$\boldsymbol{\beta}$	stress jump coefficient	$\boldsymbol{u'_f}$	dimensional velocity in the clear fluid region
Kn	Knudsen number	$\boldsymbol{u_p}$	dimensionless velocity in the porous region
$\boldsymbol{\lambda}$	molecular mean free path	$\boldsymbol{u'_p}$	dimensional velocity in the porous region
K	permeability of the porous material	$\boldsymbol{u_i}$	dimensionless steady interfacial velocity
d	interfacial position	$\boldsymbol{u_t}$	transient velocity
F	tangential momentum accommodation coefficient	$\boldsymbol{u_{ti}}$	transient interfacial velocity
$\boldsymbol{\upsilon_e}$	kinematics viscosity of the fluid for the porous domain	**P**	dimensionless pressure gradient
$\boldsymbol{\upsilon_f}$	kinematics viscosity of the fluid	$\boldsymbol{\frac{\delta p'}{\delta z'}}$	dimensional pressure gradient
$\boldsymbol{y}$	dimensionless y-coordinate	$\boldsymbol{\beta_v}$	dimensionless variable

1. Introduction

Fractional Calculus begins when the inventor of calculus Leibnitz received a letter from his colleague L'Hopital about what will happen if $\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}}$ is considered? In his reply he said a paradox of the form $\sqrt{dx{:}\,x}$ could be obtained which might paved ways for important outcomes in the future. Fractional Calculus is an area that handled arbitrary order of derivatives and integrals. In fractional Calculus, the fractional derivatives $\frac{d^{\alpha}y}{dx^{\alpha}}$ of arbitrary y are obtained by replacing $n = 0, 1, 2, \ldots$ from integer derivatives $\frac{d^{n}y}{dx^{n}}$. This fractional order (α) might be negative fraction, positive fraction or complex number.

The idea of fractional calculus was neglected during the entire 17th century up to the early 20th century, but its importance was discovered during the last three decades by scientist in different fields such as mathematics, physics, chemistry, dynamical problems, waves oscillations, electrical network etc. One of the advantages of fractional calculus are its solutions of fractional order differential equations revealed a better real-life phenomenal when compared to integer solutions obtained from integer differential equations. This could be attributed to the fact that fractional derivatives give global explanations on activities over an interval instead of a point in the case of one variable whereas integer derivatives only give descriptions of local activities at a particular point. In reality, real-life situations could happen around a particular point instead of a given point [1]. Many scientists and engineers have developed interest toward the application of fractional differential equations recently, these equations consist of fractional derivatives or fractional integrals due to its applications in different disciplines such as biology, chemistry, physics and so on. They are usually applied to porous media, random walks with memory, dynamics of complex material, dynamical systems that consist of dynamical chaotic behavior and quasi-chaotic dynamical systems [2].

Recently, the applications of fractional time derivatives in modelling fluid flow thorough porous medium have attract the attentions of so many researchers in the field of fluid mechanics. The governing equations which are in fractional order are solved analytically to obtained a fractional order solution. Hamid et al. [3] explained that for little values of time (t), while increase in the fractional parameter (α) the velocity of the fluid increases but after some critical values of time tc the behavior is reverse and also discovered that heat transfer increases as increasing the nanoparticles volume fraction parameter. Ali et al. [4] noticed that from the graphical results, the fractional couple stress nanofluid (CSNF) model described more realistic feature of the velocity distribution better than the classical CSNF model and further realized that increase in the temperature profile and concentration profile is

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observed by increasing value of α for the small values of τ. Opposite effect of α is noticed for the large values of τ. Saqib et al. [5] they explained that the numerical results obtained indicated that the fractional parameters significantly affect the temperature and velocity fields. It is noticed that the temperature field increased with an increase in the fractional parameter. Whereas the effect of fractional parameters is opposite on the velocity field near the plate. In, another research work, [6] said that fractional solutions for temperature and velocity fields are more general, reliable, and flexible, with memory and heredity properties that can be numerically reduced for any values of $0 <\propto\leq 1$. The velocity profile increases with increased permeability of the porous medium and thermal Grashof number, due to the improvement in the velocity boundary layer. Atangana and Bildik [7], said the numerical simulations show that the fractional order derivative plays an important role in the simulation process. In addition, they compare the analytical solution with experimental data to access the accuracy of the fractional groundwater model and concluded that analytical solution was in perfect agreement with the experimental data.

There are numerous definitions of fractional derivatives models, thus the available literatures have provided us with Riemann-Liouville fractional derivative model, the Caputo (C) fractional derivative model, the Weyl fractional derivative model and the newly presented definition of Caputo-Fabrizio time fractional derivative model in 2015 with local kernel that provided two expressions for the space and time variable [8]. The goal of this research is to investigate the application of fractional derivatives (in Caputo fractional time derivatives sense) to fluid flow in a horizontal microchannel filled with porous materials to obtain the solutions in fractional order, which were also utilized in [9-13]. The Caputo fractional time derivative model is used to derived the solution of velocity profile of the flow when the driven force is induced by pressure gradient subjects to velocity gradient boundary condition of homogeneous differential equations.

This research work has laid emphasis on [14] which studied unsteady flow in a horizontal parallel-plate microchannel filled with a constant uniform porous material where they neglect the inertial effects in the porous region which is not in the literature. It is also utilized the Brinkman-extended Darcy law to model the fluid flow in the porous layer, while for the clear fluid region, the Stokes equation was used to model the flow. The driven force that influenced the flow formation within the microchannel is called pressure driven force.

2. Mathematical Analysis

Ghadle et al. [15], reported that for m to be the smallest integer that exceed α, the Caputo fractional time derivative operator of order $\alpha > 0$ is defined as;

$$D_t^{\alpha} u(x,t) = \begin{cases} \dfrac{1}{\Gamma(m-\alpha)} \displaystyle\int_0^t (t-\tau)^{m-\alpha-1} \dfrac{\partial^m}{\partial t^m} u(x,\tau)\ d\tau\,, \quad for\ m-1 < \alpha \leq m, \\ \dfrac{\partial^m}{\partial t^m} u(x,\tau)\ for\ \alpha = m\,, \qquad m \in N \end{cases} \tag{1}$$

Consider the fully developed unsteady laminar fluid flow in horizontal microchannel parallel-plates filled with uniform porous medium. The flow is assumed to be in the x-direction which is taken horizontally along the channel plates which are H distance apart and the y-axis is taken normal to the plates. At $t \leq 0$, the fluid is at rest with initial velocity $u(y,0) = 0$, for all y. For $t > 0$, the velocity $u(0,t) = 0,\ at\ y = 0$ and becomes $u(H,t) = 0, at\ y = H$.

The geometry of the system under consideration in this present study is shown schematically in Figure 1.

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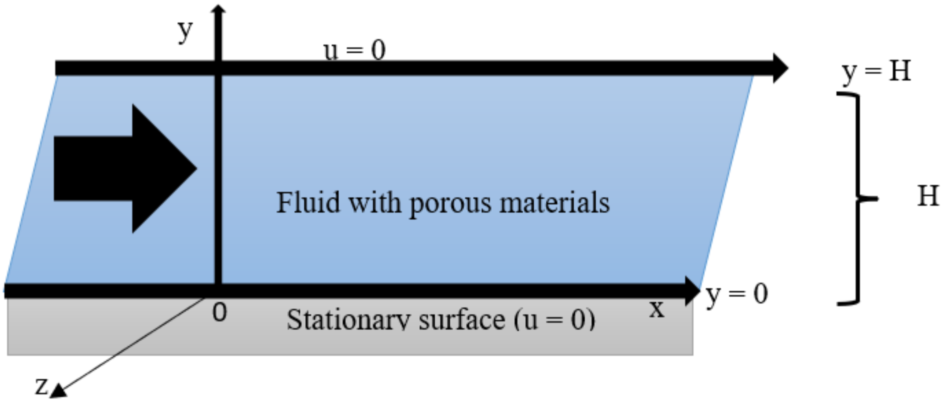

Fig. 1. Geometry of the problem.

The governing equation for the flow is given as;

$$D_t^{\alpha} u(x,t) = \gamma \frac{\partial^2 u(x,t)}{\partial y^2} - \frac{u(x,t)}{D_a} + P \tag{2}$$

with its initial and boundary condition as;

$$D_t^{\alpha}\ t \leq 0: u(y,t) = 0\ for\ all\ y u(x,t) = \gamma \frac{\partial^2 u(x,t)}{\partial y^2} - \frac{u(x,t)}{D_a} + P \tag{3}$$

$$t > 0: \begin{cases} u(y,t) = +\beta_v Kn \dfrac{du(y,t)}{dy}, & at\ y = 0 \\ u(y,t) = -\beta_v Kn \dfrac{du(y,t)}{dy}, & at\ y = H \end{cases} \tag{4}$$

where the non-dimensional parameters are given as;

$$\begin{cases} u = \dfrac{u'}{U}, \quad y = \dfrac{y'}{H}, \quad d = \dfrac{d'}{H} \\ \gamma = \dfrac{v_e}{u}, \quad D_a = \dfrac{k'}{H^2}, \quad P = -\dfrac{1}{\rho}\dfrac{H^2}{Uu}\dfrac{dP}{dx} \\ t = \dfrac{t'v}{H^2}, \quad \beta_v = \dfrac{2-F}{F}, \quad Kn = \dfrac{\lambda}{H} \end{cases} \tag{5}$$

where (Da) is the Darcy number, (Kn) is the Knudsen number, (P) is the non-dimensional pressure gradient and β_v is a non-dimensional variable. The physical quantities used in Equation (5) are defined in the nomenclature. By introducing the Laplace transform of the non-dimensional velocity of the flow $\overline{U}(s,y) = \int_0^{\infty} u(t,y)e^{-st}dt$ (Where s is the Laplace parameter), equation (2) subject to the initial condition of Equation (3) yield;

$$\frac{d^2\overline{U}(s,y)}{dy^2} - \left(\frac{s^{\alpha}Da + 1}{\gamma Da}\right)\overline{U}(s,y) + \frac{P}{\gamma s} = 0 \tag{6}$$

while boundary condition (4) becomes

$$\frac{1}{s^2} > 0: \begin{cases} \overline{U}(s,0) = +\beta_v Kn \dfrac{d\,\overline{U}(s,0)}{dy}, at\ y = 0 \\ \overline{U}(s,H) = -\beta_v Kn \dfrac{d\,\overline{U}(s,H)}{dy}, at\ y = H \end{cases} \tag{7}$$

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Equations (6) is solved, obtaining the particular solution by the method of undetermined coefficient and using the boundary conditions (7) to get the following solution.

$$\bar{U}(s,y) = k_6 coshk_1 y + k_5 sinhk_1 y + k_2$$

where $k_1 = \sqrt{\frac{s^{\alpha}Da+1}{\gamma Da}}$, $k_2 = \frac{PDa}{s^{\alpha+1}+s}$, $k_3 = \cosh(k_1 H) + \beta_v Knk_1 sinh(k_1 H)$,

$$k_4 = sinh(k_1 H) + \beta_v Knk_1 \cosh(k_1 H),\ k_5 = \frac{k_1 k_2}{\beta_v Knk_1 k_3 + k_4} \tag{8}$$

$k_6 = \beta_v Knk_1 k_5 - k_2$,

Equations (8) which is in Laplace domain need to be inverted by Riemann- sum approximation in order to determine the velocity in time domain. Due to the difficulty in obtaining the inverse of this equation, we use a numerical means. In this method, functions in the Laplace domain "s" can be inverted to time domain as follows;

$$u(t,y) = \frac{e^{\varepsilon t}}{t}\left(\frac{1}{2}\bar{U}(\varepsilon,y) + Re\sum_{k=1}^{n}(-1)^k \bar{U}\left(\varepsilon + \frac{ik\pi}{t}, y\right)\right) \tag{9}$$

Ajibade [16], clearly explained the usefulness of equation (9) to obtained desirable result, where Re is the real part, i=√(-1) is the imaginary part, n been the number of terms used in the Riemann-sum approximation and the real part of the Bromwich contour is ε which is used to invert functions in Laplace domain to time domain.
By the use of expression (8) we obtain the skin-friction (τ̂) on the plates of the channel as follows;

$$\hat{\tau}_0 = \left.\frac{dU(s,y)}{dy}\right|_{y=0} = k_1 k_5 \tag{10}$$

$$\hat{\tau}_1 = \left.\frac{dU(s,y)}{dy}\right|_{y=1} = k_1(k_5 coshk_1 - k_6 sinhk_1) \tag{11}$$

Equations (10) and (11) which are the skin-frictions at both walls are also inverted back to the time domain with the aid of expression (9).

3. Results and Discussion

The effects of fractional time derivatives on pressure driven flow through horizontal microchannel filled with porous materials have been studied. The graphical computations for different values of $\alpha, Da, Kn,\ P\ and\ t$ have been carried out with velocity as presented in Figures 2-6 to enable us clearly observed the effect of different governing parameters on the flow.

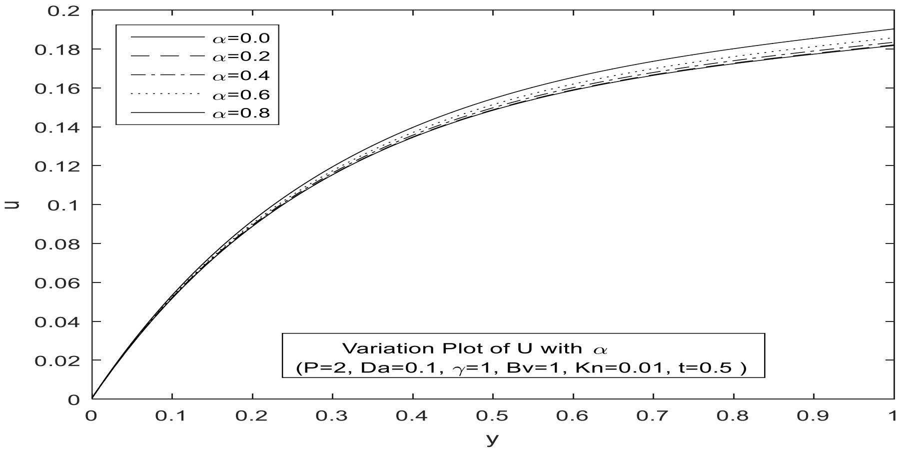

Fig. 2. Variation of fractional order (α) when the flow is induced by pressure gradient

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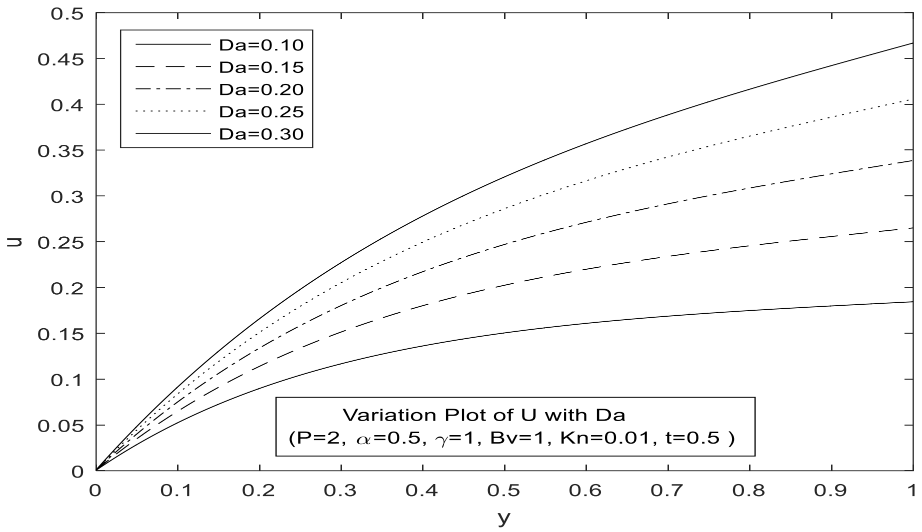

Fig. 3. Variation of Darcy number (Da) when the flow is induced by pressure gradient.

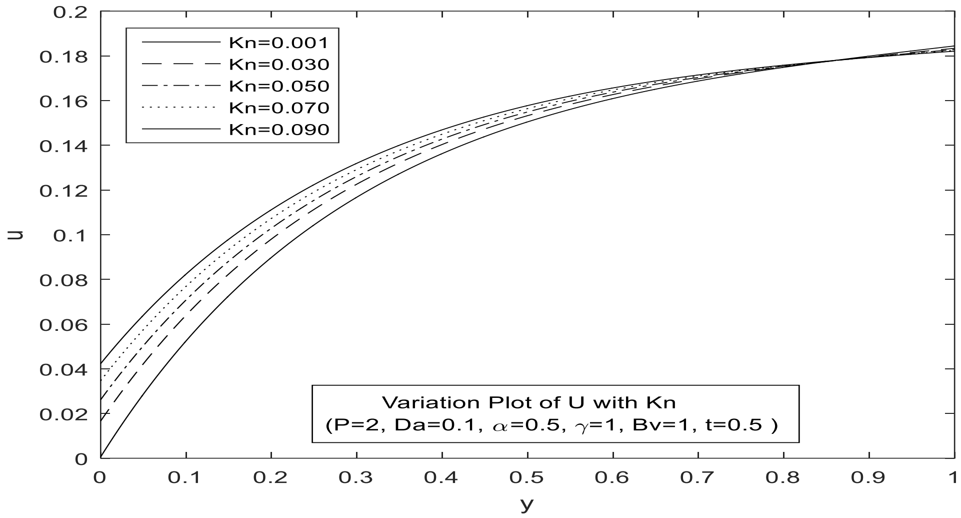

Fig. 4. Variation of Knudsen number (Kn) when the flow is induced by pressure gradient.

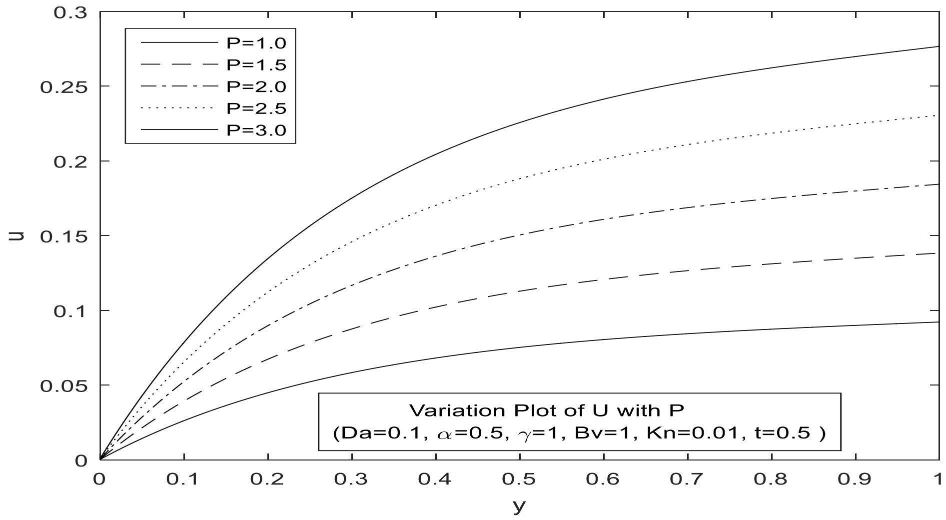

Fig. 5. Variation of pressure (P) when the flow is induced by pressure gradient.

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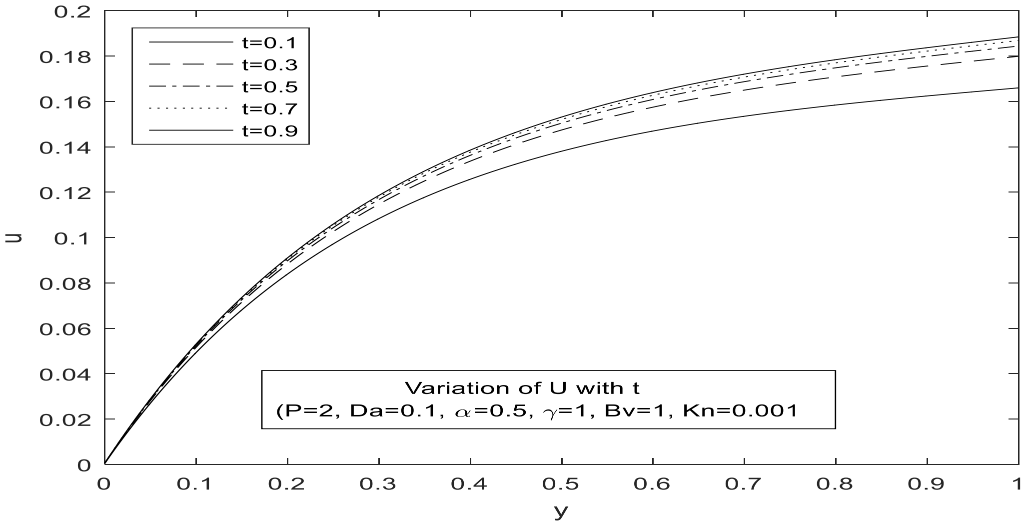

Fig. 6. Variation of time (t) when the flow is induced by pressure gradient.

At Figure 2, it can be observed that increasing the fractional order α between the interval of $0 \leq \alpha < 1$ increases the velocity while at $\alpha \geq 1$, it displays an irregular flow pattern. Figure 3 displays the flow of increasing Darcy number (Da) increases the velocity of the fluid. This implies that increasing the porousity enhances the fluid flow within the microchannels. Figure 4 also reveals an increase in Knudsen number (Kn) increases the flow velocity which is an indication that enlarging the width of the microchannel increases the fluid flow but converges as the flow progresses and a converse flow is observed within the channel. Similarly, Figure 5 also displays the same flow pattern as figure 3, it is clearly seen that increasing the pressure of the flow within the microchannel increases the fluid velocity which is the only driven force under consideration when negleting the movement of one of walls. At Figure 6, due to the unsteady fluid flow, the velocity is found to increase when increasing the value of time (t). This is an indication that with a constant pressure, the flow velocity through the porous medium is independent of time to come to rest or to attained steady state.

The effects of fractional order, Darcy number and Knudsen number on the skin friction at the plates are obtained with expressions (10) and (11) as shown in Table 1.

Table 1. Effects of Fractional order, Darcy number and Knudsen number on skin friction.

Da = 0.1, Kn = 0.001, γ = 1, β_v = 1, t = 0.5, P = 2			α = 0.5, Kn = 0.001, γ = 1, β_v = 1, t = 0.5, P = 2			Da = 0.1, α = 0.5, γ = 1, β_v = 1, t = 0.5, P = 2		
α	τ_0	τ_1	Da	τ_0	τ_1	Kn	τ_0	τ_1
0.1	0.0634	0.0044	0.10	0.0587	0.0031	0.001	0.0587	0.0031
0.2	0.0625	0.0041	0.15	0.0683	0.0060	0.020	0.0550	0.0028
0.3	0.0614	0.0039	0.20	0.0752	0.0087	0.030	0.0532	0.0026
0.4	0.0602	0.0035	0.25	0.0806	0.0112	0.040	0.0515	0.0024
0.5	0.0587	0.0031	0.30	0.0850	0.0134	0.050	0.0499	0.0023
0.6	0.0571	0.0028	0.35	0.0886	0.0154	0.060	0.0484	0.0021
0.7	0.0551	0.0023	0.40	0.0916	0.0171	0.070	0.0470	0.0020
0.8	0.0530	0.0019	0.45	0.0942	0.0187	0.080	0.0457	0.0019
0.9	0.0507	0.0015	0.50	0.0964	0.0200	0.090	0.0444	0.0018
1.0	0.0481	0.0012	0.55	0.0984	0.0213	0.100	0.0433	0.0017

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It can be noticed that increase in fractional order $\boldsymbol{\alpha}$ and Knudsen number Kn decreases the skin friction on both walls which implies that gradual increasing the diameter of the channel decreases the effects of the fluid molecules on the skin friction while the converse is the case for Darcy number (Da) where increase in (Da) increases the skin friction on both walls. This is an indication that increasing the Darcy number (Da) increases the permeability of the fluid flow through the porous medium.

4. Conclusion

This study investigates the effect of fractional time derivatives on unsteady fluid flow induced by pressure driven flow through horizontal parallel microchannel filled with porous medium. The influence of the governing parameters is discussed with help of the line graphs. It is observed that the increase in fractional order within some certain interval increases the velocity profile. Furthermore, the velocity profiles increase with increase in Darcy number (Da), Knudsen number (Kn), pressure (P) and time (t). However, the fluid flow on the skin friction increases with increase in Darcy number (Da) but decreases with increase in both fractional order $\boldsymbol{\alpha}$ and Knudsen number (Kn).

Authorship contribution statement for Contributor Roles Taxonomy

Muhammad Lawan Kaurangini, *Supervision, Review, Correction, Methodology, Writing.* **Isyaku Shu'aibu Abdulmumini,** *Investigation, Writing-original draft, Conceptualization, Visualization, Formal analysis*. **Umar Muhammad Abubakar,** *Writing, Correction, Review, Methodology.*

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