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Research Article

Fuzzy Modeling of non-MCDM Problems Under Indeterminacy

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Abstract

Fuzzy set theory (FST) is a popular approach for modeling the uncertainties of real-life problems. In some cases, uncertainty level of the events may not be determined surely because of some environmental factors. There are various FST extensions in the literature that consider such indeterminacy cases in modeling. Since some parts of the theories of FST extensions overlap with some others, the theories and the nature of considered scenarios must be understood well to obtain reliable results. Nevertheless, most of the studies in the literature do not conceptually analyze the nature of the uncertainty and decides an FST extension as a pre-step of the study without expressing an apparent reason. Therefore, the quality of the obtained results becomes questionable. Most of the FST extensions have been developed in line with the requirements of Multi-Criteria Decision-Making (MCDM) problem thus assumptions and limitations of these theories can cause reliability issues for the fuzzy models of the problems different from MCDM. In the scope of this study, capabilities, advantages, and disadvantages of well-known FST extensions that consider indeterminacy are conceptually analyzed and compared in line with the needs of modeling of the continuous systems, MCDM problems, and different problems from MCDM. The analysis has also been illustrated on numerical examples to make findings clear. The analysis showed that some extensions have clear advantages over others for specific scenarios. This study is an invitation to fulfill the gap in the field of fuzzy modeling of the different problems from MCDM.

Keywords: Fuzzy modeling, Fuzzy set extensions, Indeterminacy, MCDM.

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1. Introduction

Vast majority of real-world applications of engineering problems include uncertainty. However, most of the engineering techniques have been designed for certain events so their results can have reliability problem. Fuzzy set theory (FST) provided a big contribution to the literature for modeling of uncertainty. FST has been extended in different perspectives to model the uncertainties having different natures originated from their causes such as hesitation of the experts about the event. It is crucial to choose the most suitable FST extension in modeling to achieve reliable results. Zadeh [1] proposed the FST to deal with the problems in which the source of uncertainty is caused by the lack of clear criteria of class membership rather than the existence of random variables. For an Ordinary Fuzzy Set (FS), certainty level about an event is represented with membership degree (MD) concept. While MD is getting bigger, the certainty of the event increases.

In some cases, it is difficult to decide the MDs of the elements because of some factors such as lack of expertness about the event or system. In such cases, uncertainty level cannot be well-defined. This situation is named as “Indeterminacy” scenario and Indeterminacy Degree (IDD) represents the level of lack of information about MD. Atanassov [2] extended the FST with the name Intuitionistic FS (IFS) to provide better modeling of cases that include lack of information about MD. In IFS theory, IDD is decided based on MD and Non-Membership Degree (NMD). In some cases, such as collecting data from multiple sources can cause inconsistency between MD and NMD. This should also be handled in modeling to reach reliable results for these scenarios. For this aim, Smarandache [3] generalized IFSs as Neutrosophic Set (NS) in a theoretical perspective by allowing independent MD, NMD and IDD to give ability to model with inconsistent data. However, logical meaning of “independency of IDD” is a question mark for most of the scenarios. Yager [4] suggested Pythagorean FS (PFS) as a theoretical extension of IFSs based on Pythagoras' theorem for the modeling of the systems with inconsistent data. For PFSs, the sum of the squares of MD and NMD is limited by one. In PFS theory, IDD is dependent on MD and NMD, but it gives ability to model the scenarios including inconsistency. However, allowed inconsistency level is limited for PFSs. To extend this limitation, Yager [5] generalized the PFS for q^th power of the MD and NMD with the name q-Rung Orthopair FS (q-ROFS). Q-ROFS theory gives more flexibility than PFS theory but deciding the best fitting q brings calculation complexity. Senapati & Yager [6] offered Fermatean FS (FFS) by changing the condition of PFSs as the sum of the 3^rd power of MD and NMD is limited by one. This approach provides a larger limitation for inconsistency level than PFS theory. These extensions give ability to model inconsistency and indeterminacy cases, but they may not always be sufficient to model the uncertainty in a reliable way. Lack of information about the event may also be caused by the refusal of the information source to provide information. Cuong & Kreinovich [7] generalized the IFSs with the name Picture FS (PcFS) by considering a new concept named “neutral membership degree” (NeMD) besides MD, NMD and IDD to give ability to model the refusal case. PcFS is capable of modeling the scenarios including indeterminacy with consistent data. If the scenario also includes inconsistency, it is not suitable for modeling. Kahraman & Kutlu Gündoğdu [8] developed Spherical FS (SFS) as a generalization of PcFSs by employing a similar approach with PFS theory to allow inconsistent data case. In SFS theory, the sum of the squares of the MD, NMD and NeMD is bounded by one.

In the literature, there are huge number of studies using FST extensions. Vast majority of these studies focus on one-time Multi-Criteria Decision-Making (MCDM) problem. For this reason, theories have been built in line with the requirements of MCDM problem. Most of the formulations do not offer sufficient capabilities for modeling of continuous systems, different problems than MCDM, and even the Decision Making (DM) problems that need sensitive calculation in decision cycle. For these types of problems, usage of the reviewed FST extensions without making novel contributions by integrating more sophisticated mechanisms brings reliability issue for the obtained results or makes sensitivity analysis dysfunctional. For example, if the variability of the collected data increases during a time, the collected data may become useless with the PFS model of a continuous system because of violating the inconsistency limitation. Even worse, if this is noticed after a while, the results will be unreliable during this period. There are reliability issues for also MCDM problems in most of the studies that use one of the reviewed FST extensions. Because, these studies do not present the motivation lying behind their FST extension preference. This prevents evaluating the compatibility of the FST extension with the scenario. For example, in the review studies conducted by Kaya et al. [9], Mardani et al. [10], and Salih et al. [11], the suitability of the scenarios to the preferred FST extensions is not considered in a conceptual perspective. Similarly, none of the studies cited in these review studies, provide a satisfactory justification for their FST extension choices and discuss the suitability of the preferred FST extension with the considered scenario from a conceptual perspective. In most of the studies such as [12], a linguistic term set including indeterminacy is pre-determined without presenting any reason, and experts choose between these terms without making any additional justification as if they are using ordinary FS during the assessment operations. This means that the indeterminacy is emerged by the initial configuration itself not by the hesitancy of the experts. It is clear that such an application will yield results having reliability issue. There are some other review studies in the literature investigating the FST extensions, but they generally focus on the mathematical aspect. For example, Kahraman et al. [13] classified the FST extensions in two groups as hesitancy dependent (IFS, PFS, FFS, q-ROFS) and refusal degree dependent (NS, PcFS, SFS) with a mathematical definition-oriented perspective and does not present a conceptual analysis. Unlike the mentioned studies, only Sevastjanov & Dymova [14] criticizes the necessity and applicability of NSs, PFSs and SFSs in a conceptual perspective but it does not present a detailed comparison.

In the scope of this study, it is aimed to conceptually analyze and compare the FST extensions that consider indeterminacy to build a guide for the selection of the suitable FST extension in fuzzy modeling of the non-MCDM problems (continuous systems and the problems different from one-time MCDM problems) by considering indeterminacy. By this way, an initial step is taken for filling the gap in the field of improving the reliability of fuzzy modeling of the different problems from MCDM. It is expected that this study will also create a motivation for studies to increase the reliability of MDCM models.

The paper is organized as follows: FST extensions including indeterminacy are analyzed and compared according to some pre-determined criteria in Section 2. The comparative analysis is summarized, opinions and findings are presented in Section 3. Conclusions and future research directions are presented in Section 4.

2. Material and Methods

Various extensions of FST have been offered to handle different type of uncertainties. Close to twenty FST extensions and their hybrid combinations have been studied in the literature. Indeterminacy is considered as a part of uncertainty in nearly half of these studies. Figure 1 shows the relations between the popular FST extensions considering indeterminacy. More than 50.000 studies were conducted since 2012 by using these FST extensions according to Google Scholar database. When the studies are analyzed in detail, it is seen that considered scenarios are too similar. For this reason, understandings the theories and the nature of scenarios is crucial to obtain reliable results. Following subsections are summarizing the theoretical backgrounds of these FST extensions.

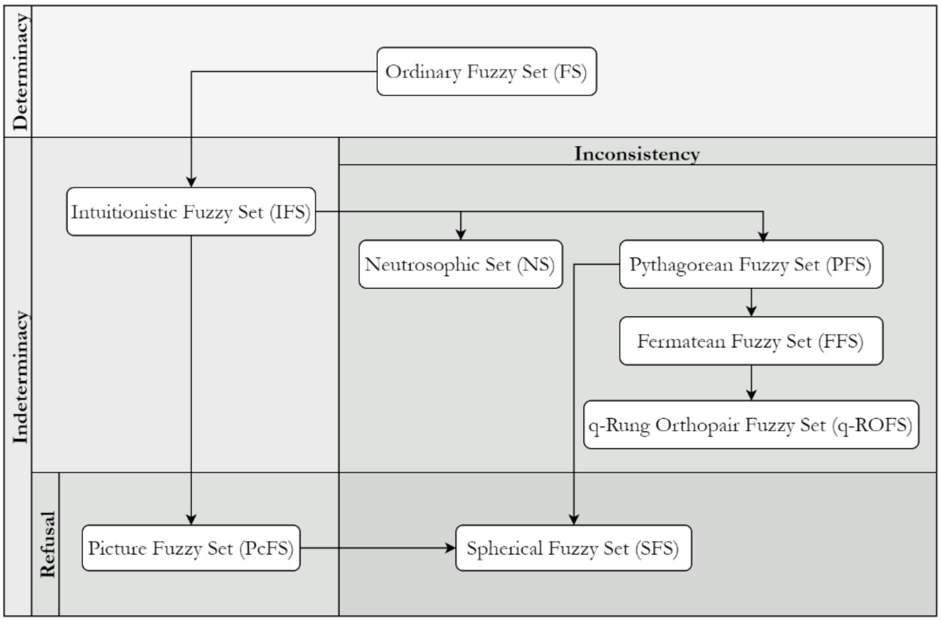

Figure 1. Relations between fuzzy set extensions that consider indeterminacy

2.1. Fuzzy Set Extensions Considering Indeterminacy

The uncertainty is modelled with the membership function (MF) concept based on a continuous variable x in [0, 1] in FST. A set element can be partially member and non-member simultaneously in FST. The level of uncertainty of the membership of a set element is represented with the term MD. If MD is high, the uncertainty is low; and if it is low, the uncertainty is high [1]. A FS (namely ordinary FS) is defined as below:

Definition 1: Let X be a given universe, $\mu_{\tilde{A}}(x) \in [0,1]$ be the MF and $\vartheta_{\tilde{A}}(x)$ be the NMF. An FS $\tilde{A}$ is defined as $\tilde{A} = \{ x, \mu_{\tilde{A}}(x) \mid x \in X \}$ and satisfies Eq. (1) [1]:

$$\mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) = 1 \tag{1}$$

MFs can have different shapes but the most popular ones are triangular and trapezoidal FSs. Figure 2 shows examples for these popular MF shapes. A triangular FS is represented with three points as (a,b,c): the limiting two of them having MD = 0 (a and c) are “support points” and third point having MD = 1 (b) is “core”. A trapezoidal FS is represented with four points as (a,b,c,d): the limiting two points a and d are the support points and the interval having MD=1 [b,c] is core. For a trapezoidal FS, every point inside [b,c] interval is a core point.

If it is hard to decide the FS shape, support points and the core, direct usage of ordinary FSs may not be possible. In most real cases, it is more practical to use interval-valued numbers as MF and NMF. These type of FSs are named Interval Valued FSs (IVFSs). An IVFS is defined as below:

Definition 2: Let X be a given universe and, $\mu_{\tilde{A}}^{-}(x)$, $\mu_{\tilde{A}}^{+}(x)$ are fuzzy subsets of X. An IVFS $\tilde{A}$ on X is defined as $\tilde{A} = \{ x, [\mu_{\tilde{A}}^{-}(x), \mu_{\tilde{A}}^{+}(x)] \mid x \in X \}$ and satisfies Eq. (2) [15]:

$$\mu_{\tilde{A}}^{-}(x) \leq \mu_{\tilde{A}}^{+}(x), \forall x \in X \tag{2}$$

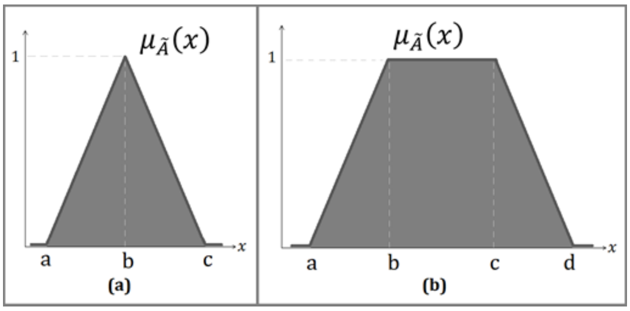

Figure 2. Membership functions; (a) triangular and (b) trapezoidal fuzzy sets

2.1.1. Intuitionistic Fuzzy Sets

In FSTs, MD and NMD are complement of each other and the sum of MD and NMD is equal to 1 for each set element. However, it may not always be possible to determine the MD and NMD values such that whose summation is equal to 1. This type of scenarios is named "incomplete information case". IFSs have been proposed for modeling the uncertainties including incomplete information case. An IFS is defined as below:

Definition 3: Let X be a given universe, $\mu_{\tilde{\tilde{A}}}(x) \in [0,1]$ be the MF and $\vartheta_{\tilde{\tilde{A}}}(x)$ be the NMF. An IFS $\tilde{\tilde{A}}$ is represented as $\tilde{\tilde{A}} = \{ x, \mu_{\tilde{\tilde{A}}}(x), \vartheta_{\tilde{\tilde{A}}}(x) \mid x \in X \}$ and satisfies Eq. (3) [2]:

$$\mu_{\tilde{\tilde{A}}}(x) + \vartheta_{\tilde{\tilde{A}}}(x) \leq 1 \tag{3}$$

In IFS theory, MD and NMD of set elements are interdependent since the value of MD limits the possible values of NMD. The difference between 1 and the sum of MD and NMD is named Indeterminacy Degree (IDD) and is calculated as shown in Eq. (4) [9]:

$$\pi_{\tilde{\tilde{A}}}(x) = 1 - \mu_{\tilde{\tilde{A}}}(x) - \vartheta_{\tilde{\tilde{A}}}(x) \tag{4}$$

IFS formulation generalizes the ordinary FS formulation. It becomes equivalent with ordinary FSs when 0 is assigned to IDD.

2.1.2. Picture Fuzzy Sets

IFS theory assumes that the lack of information about MD and NMD is caused by indeterminacy. However, it can be caused by some other factors such as refusal of the one who make assessment. Voting is a proper example of a such scenario. The voters can be grouped as: (i) vote for, (ii) vote against, (iii) vote blank, (iv) non-voting [7]. Consideration of this type of scenarios yields a new FST extension namely Picture Fuzzy Set (PcFS). A PcFs is defined as below:

Definition 4: Let X be a given universe, $\mu_{\tilde{A}}(x) \in [0,1]$ be the MF and $\vartheta_{\tilde{A}}(x) \in [0,1]$ be the NMF and, $\eta_{\tilde{A}}(x) \in [0,1]$ be the neutral membership function (NeMF). A PcFS $\tilde{A}$ is represented as $\tilde{A} = \{ x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x), \eta_{\tilde{A}}(x) \mid x \in X \}$ and satisfies Eq. (5) [7]:

$$\mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) + \eta_{\tilde{A}}(x) \leq 1 \tag{5}$$

Refusal degree (RD) is calculated as shown in Eq. (6) [7]:

$$\rho_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \vartheta_{\tilde{A}}(x) - \eta_{\tilde{A}}(x) \tag{6}$$

2.1.3. Neutrosophic Sets

In some cases, such as providing information from multiple sources, the sum of MD and NMD can exceed 1 and the main condition of FST presented in Eq. (1) is violated. This scenario is named as "inconsistent information case". If inconsistency is available on data and it is planned to use ordinary FSs or IFSs in modeling, the data must be converted by using an approximation approach or the data must be recollected until reaching consistent data. However, it may not be possible to collect suitable data even if it is tried again and again. An assumption or a change in data collection procedure may be needed to collect suitable data but this generally means loss of information. If the transformation is made on the collected data, it will also cause loss of information. Both these approaches can reduce the model's reliability. To avoid such transformations, NS theory was suggested in the literature. NS is the generalization of IFS. NS gives ability to set values for membership, non-membership, and indeterminacy independent from each other. This leads up the ability of modeling with inconsistent data.

The terminology for NSs is different than IFS terminology. The MD is named "truthiness" and NMD is named "falsity". As a characteristic feature of NS theory, the indeterminacy is handled as a separate term. An NS is defined as below:

Definition 5: Let $t \in [0,1]$ be truthiness, $f \in [0,1]$ be falsity and, $i \in [0,1]$ be indeterminacy. A NS $\ddot{\tilde{A}}$ is defined as $\ddot{\tilde{A}} = (t, i, f)$ and satisfies Eq. (7) [3]:

$$0 \leq t + i + f \leq 3 \tag{7}$$

Some problems may not be suitable for modeling with inconsistent data. For such cases, loss of information is inevitable. For such problems, the data can be normalized by dividing each of the terms with total of the terms to satisfy Eq. (8) [16]. After the normalization, the model becomes conceptually near-equivalent to IFS model.

$$t + i + f = 1 \tag{8}$$

2.1.4. Pythagorean Fuzzy Sets

For the considered problem, if the inconsistency is always occurred under a specific limit, PFS can be used for modeling the event. A PFS is defined based on two concepts: support for membership and support against membership.

Definition 6: Let X be a given universe, and $\theta(x) \in [0, \pi/2]$ be a radian angle. The support for membership ($A_Y(x)$) and the support against membership ($A_N(x)$) for a PFS $\tilde{A}$ are defined as in Eq (9) [4].

$$A_Y(x) = r(x) \times \cos\big(\theta(x)\big),\ A_N(x) = r(x) \times \sin\big(\theta(x)\big) \tag{9}$$

The strength of commitment ($r(x) \in [0,1]$) and the direction of commitment ($d(x) \in [0,1]$) are defined as shown in Eq. (10) [4]:

$$r(x) = \sqrt{\left(A_Y^2(x) + A_N^2(x)\right)},\ d(x) = \frac{(\pi - 2 \times \theta(x))}{\pi} \tag{10}$$

Eq. (11) shows the definition of a PFS $\tilde{A}$ with the terminology and symbols of IFS theory.

$$\tilde{A} = \{\, x, \mu_{\tilde{A}}(x) = A_Y(x), \vartheta_{\tilde{A}}(x) = A_N(x) \mid x \in X \,\}, \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) \leq 1 \tag{11}$$

Depending on this definition, IDD of an element $x \in X$ is obtained as shown in Eq. (12):

$$\pi_{\tilde{A}}(x) = \sqrt{1 - r^2(x)} = \sqrt{1 - \left(\mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x)\right)} \tag{12}$$

2.1.5. Fermatean Fuzzy Sets

Modeling with PFS is possible if and only if the inconsistency of data is under a limit. If the inconsistency violates Eq. (11) for some of the set elements, PFSs cannot be used. The general approach may be using NSs. However, if the independency of MD, NMD and IDD is not a desired circumstance, NS may not be suitable. FFS is a similar FST extension with PFS but its inconsistency limit is larger than PFS. An FFS is defined as follows:

Definition 7: Let X be a given universe, $\alpha_{\tilde{\mathcal{F}}}(x) \in [0,1]$ be MD, and $\beta_{\tilde{\mathcal{F}}}(x) \in [0,1]$ be MD for a set element $x \in X$. An FFS $\tilde{\mathcal{F}}$ is represented as $\tilde{\mathcal{F}} = \{ x, \alpha_{\tilde{\mathcal{F}}}(x), \beta_{\tilde{\mathcal{F}}}(x) \mid x \in X \}$ and satisfies Eq (13) [6].

$$0 \leq \alpha_{\tilde{\mathcal{F}}}^3(x) + \beta_{\tilde{\mathcal{F}}}^3(x) \leq 1 \tag{13}$$

Thus, IDD of an element $x \in X$ is found as shown in Eq. (14) [6]:

$$\pi_{\tilde{\mathcal{F}}}(x) = \sqrt[3]{1 - \left(\alpha_{\tilde{\mathcal{F}}}^3(x) + \beta_{\tilde{\mathcal{F}}}^3(x)\right)} \tag{14}$$

2.1.6. Q-rung Orthopair Fuzzy Sets

q-ROFS is the generalization of the IFS, PFS, and FFS for q^{th} power of the MD and NMD. The comprehensiveness of inconsistency increases while q is getting bigger. A q-ROFS is defined as follows:

Definition 8: Let X be a given universe, q be a positive real number, $A^+(x) \in [0,1]$ be the degree of support for membership, and $A^-(x) \in [0,1]$ be the degree of support for non-membership. A q-ROFS $\tilde{A}$ is represented as $\tilde{A} = \{ x, A^+(x), A^-(x) \mid x \in X \}$ and satisfies Eq (15) [5].

$$0 \leq A^+(x)^q + A^-(x)^q \leq 1 \tag{15}$$

q-ROFS can be rewritten with the same terminology with IFSs as shown in Eq. (16):

$$\tilde{A} = \{ x, \mu(x), \vartheta(x) \mid x \in X \}, 0 \leq \mu(x)^q + \vartheta(x)^q \leq 1 \tag{16}$$

Accordingly, IDD is calculated as in Eq. (17) [5]:

$$\pi_{\tilde{A}}(x) = \sqrt[q]{1 - (\mu(x)^q + \vartheta(x)^q)} \tag{17}$$

2.1.7. Spherical Fuzzy Sets

PcFS does not allow modeling the uncertainties including inconsistent data. SFS is the generalized version of PcFS giving ability to model inconsistent data scenarios. The generalization is made with a similar logic with PFSs. A SFS is defined as below:

Definition 9: Let X be a given universe, $\mu_{\tilde{A}}(x) \in [0,1]$ be the MF, $\eta_{\tilde{A}}(x) \in [0,1]$ be the NeMF, $\vartheta_{\tilde{A}}(x) \in [0,1]$ be the NMD. A SFS $\tilde{A}$ is represented as $\tilde{A} = \{x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x), \eta_{\tilde{A}}(x) | x \in X \}$, and satisfies Eq. (18) [8]:

$$0 \leq \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) + \eta_{\tilde{A}}^2(x) \leq 1 \tag{18}$$

Accordingly, RD of an element $x \in X$ is yielded as shown in Eq. (19):

$$\rho_{\tilde{A}}(x) = \sqrt{1 - \left(\mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) + \eta_{\tilde{A}}^2(x)\right)} \tag{19}$$

2.2. Conceptual Comparison of Fuzzy Set Extensions

The mentioned FST extensions in previous section are developed in line with the needs of Multi-Criteria Decision-Making (MCDM) problem. MCDM problems require single time modeling, and the process is finalized once a decision is made. If a new decision is needed and the environmental conditions are changed, the decision process should be rebuilt. However, some of the Decision Making (DM) problems and some other engineering problems are constructed for continuous systems. For example, Acceptance Sampling Plan (ASP) is a DM problem that assumes a continuous incoming item flow in decision process. Thence, the variability of the defectiveness of the incoming items should be considered by the decision process. The main purpose of this study is to assess the capabilities, advantages and disadvantages of the FST extensions that consider indeterminacy in a conceptual perspective for the modeling of continuous systems. This analysis can guide for the selection of the most appropriate FST extension in modeling of continuous systems.

Understanding the scenario has vital importance for successful modeling. FST extension must be decided after understanding the scenario and the causes of uncertainty well. The uncertainty of the considered scenario can be caused by the environment or the system itself. Hence, different FST extensions may be more suitable in different environments for the same problem. Essentially, understanding of the scenario is not sufficient for a high-quality modeling. FST extensions should also be well-understood. In this study, FST extensions have been analyzed based on the following criteria: (i) ease of calculation and implementation, (ii) scenario comprehensiveness, (iii) applicability in real cases and (iv) number of assumptions to obtain a useful guide for selection of the most appropriate FST extension in modeling of continuous systems. The rest of this section compares the FST extensions based on these criteria.

2.2.1. Ease of Calculation and Implementation

When an engineering technique is modified to handle the uncertainty, the complexity of the formulations increases. As the formulation becomes more complex, it requires more complex input data. The system to be modeled and the environment must give ability to collect suitable complex data to use the complex formulation efficiently. Since it may not be possible to collect complex data in some real cases, complex formulations may not be practicable in some scenarios. From this point of view, ordinary FS is the most advantageous alternative in terms of ease of implementation and calculation. On the other hand, using too simpler FST extension may cause reliability issues. For this reason, possibly complex but applicable FST extension should be selected. Even if the scenario seems suitable for modeling with a more complex FST extension, possible simplifications without loss of information should be made. By this way, the ease of calculation and implementation is provided without facing reliability issues. If the scenario allows conversion to ordinary FSs with ignorable loss of information, it is better to use ordinary FSs for gaining simplicity. Similarly, if the scenario allows conversion to IFS with neglectable loss of information, NS, SFS, PcFS, PFS, FFS and q-ROFS should be avoided. If indeterminacy and inconsistency is available together and both cannot be eliminated without loss of information, NS may be the best FST extension preference in terms of calculation simplicity. On the other hand, if linguistic approach is used, NS can harm the interpretability because of additional independent term representing indeterminacy. Because the logical meaning of independency of indeterminacy term can cause confusion for some cases. PFS, FFS and q-ROFS may be preferred if the independency of MD, NMD and IDD is not a desired case. Among these, the FST extension that is closer to ordinary FS should be selected to provide ease of calculation and implementation and reduce loss of information. Because, when a looser inconsistency limit is decided, indeterminacy term will be calculated bigger, and the accuracy of the results will be reduced. Therefore, if the inconsistency limit is appropriate, the first choice should be PFS, then FFS and so on. If the scenario allows conversion to IFS with neglectable loss of information, it may be better to use IFS. It should be noted that one should be cautious while using PFS, FFS, SFS or q-ROFS for modeling of continuous systems without using a mechanism guaranteeing to collect data suitable with the inconsistency limitations. Otherwise, the system can produce values violating inconsistency limit and the model can have reliability issue.

If the scenario includes refusal case and it cannot be eliminated, the elimination of inconsistency should be evaluated for simplicity. If it is possible, PcFS becomes usable.

2.2.2. Scenario Comprehensiveness

Some of the FST extensions are the generalization of others. Generalization means covering them with some extra scenarios. For example, SFS is the most comprehensive FST extension. It may seem better to choose the most comprehensive FST extension in modeling but the complexity in calculation is the cost of this preference. If and only if the complexity is acceptable and this selection is value added for the further analyses, it will be better to select the comprehensive one. For example, NS is the generalizations of IFS. If the interested scenario includes inconsistent data case and the calculation complexity is acceptable, NS can be preferred. Otherwise, it would be better to use IFSs. Similarly, if the refusal case is a valid for the problem, PcFS can be preferred instead of IFSs.

With this point of view, NS is more advantageous than PFS, FFS, and q-ROFS. Modeling with PFS is possible if and only if the inconsistency level of data is always occurred under a limit. If the inconsistency violates Eq. (11) for some of the set elements, PFS cannot be used. As presented in previous section, FFS, and q-ROFS are FST extensions developed by a similar logic with PFS but with a greater inconsistency limit. Unfortunately, the risk of violating the validity condition (Eqs. (13) and (16)) is possible when a continuous system is modeled by using them. If they are not supported with a data collection mechanism limiting the inconsistency, the more reasonable approach is using NS. It should be noted that usage of NS may not be suitable if the independency of MD, NMD and IDD is not a desired circumstance or brings confusion and complexity.

2.2.3. Applicability in Real Cases

The applicability of an FST extension in real cases is related with the real-life availability of the scenario. If the main concern of the FST extension is not a common real issue, the applicability will be low. Each additional parameter brings complexity to the model, and the application may sometimes fail due to the complexity. If some parameters such as refusal are negligible, it would be better to simplify the scenario.

Usage of PFS, FFS, SFS, and q-ROFS in modeling of continuous systems is a bit problematic. Because, if there is no mechanism guaranteeing to collect data suitable with the inconsistency limitations, the collected may become incompatible with the model. In a such case, a transformation is needed, but loss of information is faced because of transformation operation. Another problem related to reliability is that the unsuitability of the scenario with the model may not be understood for a while during the application. If the system starts producing unsuitable data after a while, the obtained results may become garbage. Thus, it would be better to avoid using PFS, FFS, and q-ROFS while modeling continuous systems. Moreover, they do not offer more capabilities than NS and bring calculation complexity and risk of violating the validity condition because of high inconsistency. If there is an ability to make system enhancement to avoid inconsistency, it should be the first thing to be done. After such an enhancement, IFS will be suitable for the scenario. Similar comparison can be made between SFS and PcFS.

2.2.4. Number of Generalizing Assumptions

Each FST extension is built on some assumptions. If an FST extension is preferred without understanding its assumptions, there is a risk of applying it to a wrong scenario and the reliability problem can be occurred for the obtained results. If an FST extension having fewer generalizing assumptions is preferred, the probability of facing with the reliability problem decreases. Generally, simpler FST extensions have more generalizing assumptions. For example, ordinary FS assumes that MD can be decided surely for all set elements. However, it may not be the case for the scenarios including human factor. Because of some factors such as hesitation of experts while assessing the uncertainty can cause lack of information about MD. IFS, NS, PFS, FFS and q-ROFS assume that the only cause of lack of information about MD is indeterminacy, but this is also a generalizing assumption. PcFS and SFS narrow the generalization of this assumption and propose the refusal concept as a new source of the lack of information about MD.

FST extensions have also assumptions about the inconsistency. Ordinary FS, IFS and PcFS have designed for the scenarios that do not produce inconsistent information. NS accepts inconsistent data and assumes that the root cause of the inconsistency is collecting data from multiple sources. For this reason, it considers MD, NMD and IDD as independent terms and allows unlimited inconsistency. However, the theory misses the logical paradoxes for the opposite asymptotic limits ($t = 1, i = 1, f = 1$) and ($t = 0, i = 0, f = 0$) [14]. PFS, FFS, q-ROFS and SFS assume that the nature of the inconsistency has a pattern with an exponential mathematical expression so it will always be occurred under a limit. In fact, it is hard to prove that the inconsistency of the system has a mathematical pattern in long run for the continuous systems.

3. Results and Discussion

Both ordinary FS and FST extensions have some advantages over the others. Ordinary FS has advantages in terms of ease of implementation and calculation. On the other hand, FST extensions are more powerful in terms of scenario comprehensiveness. The best preference is to select the FST extension having possibly minimum number of generalizing assumptions and maximum simplicity. The explanation given in the previous section shows that there is a trade-off between these two. Thus, deciding an optimal FST extension requires deep understanding of the theories and the considered scenario. Table 1 shows the summary assessment of the FST extensions based on some factors.

Table 1. Comparison of FST extensions based on pre-determined criteria

FST Extension	Applicability In Real Cases	Ease of Calculation	Number of Generalizing Assumptions	Scenario Comprehensiveness
FS	High	High	High	-
IFS	High	Medium	Medium	Indeterminacy
NS	Medium	Medium	Low	Indeterminacy + Inconsistency
PcFS	Medium	Low	Low	Indeterminacy + Refusal
SFS	Low	Very Low	High	Indeterminacy + Refusal + Limited Inconsistency
PFS	Low	Low	High	Indeterminacy + Limited Inconsistency
FFS	Low	Low	High	Indeterminacy + Limited Inconsistency
q-ROFS	Low	Low	High	Indeterminacy + Limited Inconsistency

There are some limitations about FST extensions. IFS theory mainly attends to MD and NMD. Indeterminacy arises from the lack of information about MD and NMD. However, lack of information about MD and NMD may be caused by some other factors from indeterminacy in some scenarios. For example, some experts can refuse to make an assessment. Modeling with IFS can bring reliability problem for such scenarios. Disregarding the refusal case is an important limitation of IFSs. PcFS may be preferable alternative for such scenarios. The meaning of the word "refusal" is rejecting to make an assessment. Cuong [7] gives voting example for explaining the meaning of the term by dividing the voters into four groups: voting for a party, voting blank, voting against a party, refusing to vote. Representing the fourth group in some studies can be meaningful and PcFS gives useful results. However, it should not be ignored that even for the same voting example, the same data for another study may mean missing information depending on the research question.

NS theory looks like a good option to model the scenarios including both indeterminacy and inconsistency. Unlike PFS theory, it does not put a limit for inconsistency, but some discussions are available on the independency of truthiness, indeterminacy, and falsity terms. NS theory is seemed mathematically allows this independency, but paradoxes occur for the limit values of the terms: $t = 1, i = 1, f = 1$ and $t = 0, i = 0, f = 0$ [14]. If the data for the modeled problem is generally emerged near the limits, one should be cautious about the obtained results.

Since PFS, FFS, and q-ROFS are theoretical extensions of IFS, there are some discussions in the literature about the applicability of them in real case applications. Sevastjanov et al. [14] ask these questions for PFS: "what $\mu_{\tilde{A}}^2$ means in a natural language?" and "for which superiority was $\mu_{\tilde{A}}^2$ chosen over other powers such as $\mu_{\tilde{A}}^3$ and $\mu_{\tilde{A}}^4$?". In addition, how to ensure that Eq. (11),(13),(16) are satisfied in all cases in real life applications is also an issue to be considered. Another issue about PFS, FFS, and q-ROFS is that the indeterminacy is produced synthetically depending on the gap between 1 and the sum of the qth (2 for PFS, 3 for FFS) power of the terms. Which means that the greatness of IDD is directly dependent on initial model configuration. For example, assume that the sum of the 2nd power of MD and NMD exceed 1 but 3rd power of MD and NMD does not exceed 1. The one who models this problem can think that using FFS is the best choice. However, another people can decide to model the problem by using q-ROFS and can use a non-integer value such as 2.75 that does not violate the main condition of q-ROFS presented in Eq. (15). Similarly, If FFS is preferred for a problem instead of PFS, the greatness of the IDD dramatically increases just because of this modeling decision. Here is the question to answer: "Does the model design really affect the hesitancy of the experts?". This issue is what we named as "synthetically production of indeterminacy". This can be concluded that the indeterminacy is subjective and unreliable in PFS, FFS, and q-ROFS. Even though the greatness of IDD may not cause a huge loss of information for MCDM problems and the ranking of alternatives can be done with an ignorable error risk, it may cause reliability issue for the fuzzy models of non-MDCM problems requiring sensitivity calculation. If the problem formulation uses IDD directly in calculations and the greatness of IDD highly affects the obtained results, the obtained results will not be reliable. Acceptance Sampling Plan (ASP) is a good example for this type of problems. If the inconsistency is inevitable case and the "independency of indeterminacy term" is not a desired circumstance, it can be considered to use an advanced data collection procedure such as proposed in [17] that scales the indeterminacy in a flexible way suitable with the main condition of PFS given in Eq. (11). Similar critics can be done for SFS by considering the greatness of RD by considering Eq. (19).

Table 2 shows the obtained IDD values of IFS, PFS, q-ROFS, and NS for given 9 evaluations. The first 6 evaluations are consisting of MD and NMD while the remaining 3 of them consist of MD, NMD, and IDD. The evaluations are ranked by using the Score and Accuracy functions suggested by [18], [4], [6], [5], [19]. Results show that greatness of IDD causes little loss of information between PFS and FFS while ranking fuzzy evaluations. Amount of loss of information increases for q-ROFS while q is getting bigger. There is a difference between the results of IFS and NS too. Table 3 includes 9 evaluations that are consist of MD, NMD, and IDD and the obtained ranking results for NS, PcFS, and SFS. Results of FST extensions are different from each other in Table 3 too. In can be concluded that event for MCDM problems, the FST extension decision and the initial modeling configuration affect the obtained results.

Table 2. Comparison of produced IDD values and assessment rankings of IFS, PFS, FFS, q-ROFS, and NS for given input values

Input				Calculations															
				FS		IFS		PFS		FFS		q-ROFS (q=1.38)		q-ROFS (q=10)		NS (Normalized)			
#	MD	NMD	IDD	IDD	Rank	IDD	Rank	IDD	Rank	IDD	Rank	IDD	Rank	IDD	Rank	MD	NMD	IDD	Rank
1	0.6	0.4	-	0.00	2	0.00	5	0.69	6	0.90	6	0.34	6	1.00	4	0.60	0.40	0.00	5
2	0.7	0.3	-	0.00	1	0.00	2	0.65	1	0.86	1	0.31	1	1.00	1	0.70	0.30	0.00	2
3	0.6	0.3	-	N/A*	N/A	0.10	4	0.74	4	0.91	5	0.43	4	1.00	4	0.60	0.30	0.10	7
4	0.6	0.2	-	N/A	N/A	0.20	3	0.77	3	0.92	4	0.51	2	1.00	4	0.60	0.20	0.20	9
5	0.7	0.5	-	N/A	N/A	N/A	N/A	0.51	5	0.81	3	0.02	5	1.00	1	0.58	0.42	0.00	6
6	0.7	0.4	-	N/A	N/A	N/A	N/A	0.59	2	0.84	2	0.20	3	1.00	1	0.64	0.36	0.00	4
7	0.7	0.2	0.1	0.7	0.2	0.10	1	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.70	0.20	0.10	3
8	0.7	0.2	0.0	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.78	0.22	0.00	1
9	0.7	0.4	0.1	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	0.58	0.33	0.08	8

*N/A: Not applicable

Table 3. Comparison of produced IDD values and assessment rankings of NS, PcFS, and SFS for given input values

Input				Calculations									
				NS				PcFS			SFS		
#	MD	NMD	IDD	MD	NMD	IDD	Rank	IDD	RD	Rank	IDD	RD	Rank
1	0.6	0.4	0	0.60	0.40	0.00	4	0.00	0.00	6	0.00	0.69	6
2	0.7	0.3	0	0.70	0.30	0.00	2	0.00	0.00	3	0.00	0.65	2
3	0.6	0.3	0.1	0.60	0.30	0.10	6	0.10	0.00	5	0.10	0.73	6
4	0.6	0.2	0.2	0.60	0.20	0.20	7	0.20	0.00	4	0.20	0.75	8
5	0.7	0.5	0.2	0.70	0.50	0.20	8	N/A	N/A	N/A	0.20	0.47	7
6	0.7	0.4	0.3	0.70	0.40	0.30	9	N/A	N/A	N/A	0.30	0.51	9
7	0.7	0.2	0.1	0.70	0.20	0.10	3	0.10	0.00	1	0.10	0.68	3
8	0.7	0.2	0.0	0.70	0.20	0.00	1	0.00	0.10	2	0.00	0.69	1
9	0.7	0.4	0.1	0.70	0.40	0.10	5	N/A	N/A	N/A	0.10	0.58	4

*N/A: Not applicable

Another important point in Table 2 is the greatness of produced IDD values for PFS, FFS, and q-ROFS (Similar issue is available in Table 3 for RD values of SFS.). For example, the produced IDD value of the 4th assessment for PFS is 0.77 which is bigger than MD. For the same assessment, IDD is calculated as 0.2 (which is just one third of MD) for IFS. Similar issue is available for FFS and q-ROFS too. IDD values are getting bigger while q is increasing. In order to measure the size of the reliability issue for a non-MCDM problem that needs sensitive calculation, Acceptance Sampling Plan (ASP) formulation suggested by Işık & Kaya [20] was used and the results shown in Table 4 were obtained. As shown in Table 4, relative greatness of IDD directly affects the results of ASP problem. Acceptance probability (P_a) is found as 0 for PFS, FFS, and q-ROFS while it is found as 56% for IFS. The main cause of this difference is the theoretical background of PFS, FFS, and q-ROFS theories. Since they are developed in line with the needs of MCDM problem, they focus on ranking the alternatives. As seen in Table 2 and 3, loss of information is not so big for MCDM so the reliability issue of the results is less for MCDM problems. On the other hand, the results of these FST extensions are highly unreliable for ASP problem. The reliability issue increases while the inconsistency level of the assessments is getting smaller and the inconsistency limit of FST Extension is getting bigger.

Table 4. Acceptance sampling plan results for IFS, PFS, FFS, q-ROFS, and NS for given plan parameters

Plan Parameters					
Population Size	Sample Size	Allowed Defective Item Count	Allowed Indeterminate Item Count	MD	NMD
500	50	15	10	0.6	0.2
Results					
FST Extension	Acceptance Probability	Rejection Probability	Average Outgoing Quality	Average Total Inspection	
IFS	0.559	0.031	0.112	248.652	
PFS	0.000	0.000	0.000	499.991	
FFS	0.000	0.000	0.000	499.999	
q-ROFS (q = 1.38)	0.003	0.002	0.001	498.484	
q-ROFS (q = 10)	0.000	0.000	0.000	500.000	
NS (Normalized)	0.559	0.031	0.112	248.652	

In the literature, there is no formulation for the fuzzy ASPs considering refusal case. The ASP formulation proposed by Işık & Kaya [12] was adapted for the case of refusal. Using the acceptance probability characteristic of this adapted formulation, PcFS and SFS was numerically compared as seen in Table 5. Findings similar to those between IFS and PFS are also observed between PcFS and SFS. For the same MD, NMD, and NeMD values, the acceptance probability is found as %0.57 by modelling with SFS while it is found as %57.81 by modeling with PcFS. The table shows that the results are directly dependent on the initial modeling configurations. For ASP, the reliability issue for SFS models is caused by producing a large theoretical RD value. As shown in Table 3, a similar reliability problem does not occur for MCDM problems. Therefore, caution should be exercised when using SFS for non-MDCM problems that require precise fuzzy modeling.

Table 5. Acceptance sampling plan results PcFS and SFS for given plan parameters

Plan Parameters						
Population Size	Sample Size	Allowed Defective Item Count	Allowed Indeterminate Item Count	MD	NMD	NeMD
500	50	20	5	0.6	0.2	0.1
Results						
FST Extension		Produced RD	Acceptance Probability			
PcFS		0.10	%57.81			
SFS		0.77	%0.57			

In most of the studies in the literature, these FST extensions are used with linguistic fuzzy modeling (LFM) approach. LFM is good at interpretability, but it has weaknesses about accuracy of the results. For this reason, using LFM for the modeling of the problems requiring sensitive calculations is a challenging issue. The LFM approach should be integrated with some sophisticated mechanisms to be able to obtain accurate results and gain a capability of making an efficient sensitivity analysis. For example, ASP needs precise fuzzy modeling in decision phase, and Işık & Kaya [17] integrates the LFM with some novel mechanisms to provide sensitive and accurate results. As another example problem type, investment analysis problems in engineering economy also requires sensitive calculations in decision process.

4. Conclusion

Most of the modern engineering techniques are designed for certain events. However, the real-life problems are generally including uncertainty. FST is one the most popular approaches used to reflect the uncertainties of real-life to engineering models. The uncertainty can be caused by various factors in real world applications. The nature of the uncertainty changes depending on these factors. Several FST extensions have been offered for better modeling of uncertainties having different natures. Lack of expertness about the modeled scenario, human hesitancy while making assessment are some example factors affecting the nature of uncertainty. These types of factors can cause indeterminacy about the uncertainty level of the events. There are various FST extensions that consider indeterminacy in modeling. In fuzzy modeling, selection of the most suitable FST extension has key importance on the reliability of the obtained results. Since these FST extensions consider similar scenarios and some parts of the theories overlap with some other extensions, understandings the theories and the nature of scenarios is crucial to obtain reliable results.

The FST extensions considering indeterminacy are developed in line with the requirements of MCDM problem. For this reason, some of their assumptions are made by considering MCDM problems. The assumptions and the limitations of theories may cause reliability issues about the result of the fuzzy models of non-MCDM problems and continuous systems. In this study, capabilities, advantages, and disadvantages of the FST extensions that consider indeterminacy have been assessed and compared in a conceptual perspective in line with the needs of modeling of non-MCDM problems and continuous systems. The analysis showed that some extensions have clear advantages over others in terms of applicability, ease of calculation and scenario comprehensiveness. The analysis has been illustrated on numerical examples to make the findings clear. The analysis builds a preliminary step for a guiding approach for the selection of the most reliable FST extension in modeling.

As a future research direction, this study can be extended by analyzing some other FST extensions. Depending on this analysis, a guiding procedure can be proposed for choosing the most appropriate FST extension in fuzzy modeling.

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Appendix – List of Abbreviations

Abbreviation	*Statement*
ASP	Acceptance Sampling Plan
DM	Decision Making
FFS	Fermatean Fuzzy Set
FS	Fuzzy Set
FST	Fuzzy Set Theory
IDD	Indeterminacy Degree
IFS	Intuitionistic Fuzzy Set
IVFS	Interval Valued Fuzzy Set
LFM	Linguistic Fuzzy Modeling
MCDM	Multi-Criteria Decision-Making
MD	Membership Degree
MF	Membership Function
NeMD	Neutral Membership Degree
NMD	Non-Membership Degree
NMF	Non-Membership-Function

NS	Neutrosophic Set
P_a	Acceptance Probability
PcFS	Picture Fuzzy Set
PFS	Pythagorean Fuzzy Set
q-ROFS	q-Rung Orthopair Fuzzy Set
RD	Refusal Degree
SFS	Spherical Fuzzy Set