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Research Article

Investigating the K1(1270) – K1(1400) Mixing Angle via QCD Sum Rules

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Abstract

In this work, we investigated the mixing between strange axial vector mesons $K_1(1270)$ and $K_1(1400)$ by using QCD Sum Rules approach. Since these states couple to the same interpolating currents, we defined them in terms of orbital angular momentum eigen states K_{1A} and K_{1B}. By using the axial vector and tensor interpolating currents which are almost purely coupling to K_{1A} and K_{1B}, and then employing the orthogonality of the physical states, we obtained an analytical expression for the K_1 mixing angle. We performed a Monte Carlo based numerical analysis to estimate the value of the mixing angle.

Keywords: QCD Sum Rules, K_1 Mixing, Non-Perturbative QCD, Strange Meson.

1. Introduction

The Standard Model (SM) of particles was introduced in early 1960's and has been successfully explaining the properties and interactions of fundamental particles [1-7] since then. Its particle content was fully observed with the milestone discovery of Higgs boson with the observations of ATLAS and CMS experiments at Large Hadron Collider (LHC) [8,9]. Even though SM is very successful in explaining fundamental particles and their interactions, it cannot explain cosmological observations such as matter dominance and dark matter. Thus, the model is incomplete and a new physics beyond SM, i.e. new particles and interactions are required to overcome SM's deficiencies. Hints for the existence of such new particles or interactions might be seen in direct observations or their traces in SM interactions. A very recent and important observation as an example of the latter one is announced by LHCb experiment [10], which increased a huge interest and enthusiasm among particle physicists. Following previous results with low statistics [11,12] on the breaking of lepton universality, in March 2021, LHCb announced first 3.1 sigma significant lepton flavor universality breaking in $b \to s\, l^+l^-$ decays observed via $B^+ \to K^+\, l^+l^-$ semileptonic decay channel [10]. In these decays, light u quark is accompanied by an anti-beauty quark for B^+, and an anti-strange quark for K^+, and charged leptons can be pairs of electrons, muons or taus. In SM, the ratio of final state muons to electrons in a given semileptonic B decay is called R ratio and it is described by [13]

$$R_H = \frac{\boldsymbol{B}(B \to H\, \mu^+\mu^-)}{\boldsymbol{B}(B \to H\, e^+e^-)} \tag{1}$$

where H denotes the daughter hadron, and the branching ratios are calculated by integrating the differential branching ratios over the dilepton mass-squared range $q^2_{min} \le q^2 \le q^2_{max}$ [14,15]. Contrary to the SM predictions of the R ratio for K^+, which is $R_K = 1.00 \pm 0.01$ [14,15], it is measured as $R_K = 0.846^{+0.042}_{-0.039}\,^{+0.013}_{-0.012}$ in the aforementioned LHCb analysis[10]. Since $b \to s$ transition does not occur in tree level, this discrepancy from the SM predictions is considered as an absolute indication of a particle arising from a Beyond SM (BSM) theory, such as a heavy vector boson Z', running in the loop transition, or a hypothetical Lepto-Quark (LQ) enabling this transition in tree level.,

In QCD, the semileptonic $b \to s$ transitions do not occur only for pseudo-scalar Kaons, but they also happen in the decays of axial vector Kaons: $K_1(1270)$ and $K_1(1400)$, which have the same quark content. In this scenario, $B\ (\bar{B}) \to K_1(1270{,}1400)l^+l^-$ transitions[1] should be investigated, where pseudoscalar and neutral B meson consists of $b\bar{d}$($b\bar{u}$ or $\bar{u}b$ for positive and negative charged ones), and K_1 states are the axial vector members p-wave singlet and triplet with quark content $s\bar{d}$($s\bar{u}$ or $\bar{u}s$ for positive and negative charged ones). In Particle Data Group listings (PDG) [16], $I=1/2$, axial vector strange mesons $K_1(1270)$ and $K_1(1400)$ are listed with the following masses, $m_{K_1(1270)} = 1253 \pm 7\ MeV$ and $m_{K_1(1400)} = 1403 \pm 7\ MeV$, respectively. Since decays of $K_1(1270)$ and $K_1(1400)$ to $K\rho$ and $K^*\pi$ final states are not observed to be equal, and the lighter (heavier) is observed to decay in to $K\rho$ ($K^*\pi$) more, a large mixing between pure orbital angular momentum and G-parity eigen states is proposed [17]. In SM, a real hadron should be represented

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[1] For brevity, $K_1(1270{,}1400)$ is used as a short hand notation for $K_1(1270)$ and $K_1(1400)$, such as $B\ (\bar{B}) \to K_1(1270{,}1400)l^+l^-$ means $B\ (\bar{B}) \to K_1(1270)l^+l^-$ and $B\ (\bar{B}) \to K_1(1400)l^+l^-$.

in terms of physical mass eigen states. However, K_1 states appearing in the quark model are ${}^1P_1\,(1^{+-})$ and ${}^3P_1\,(1^{++})$, which are p-wave excited orbital angular momentum states. These states are named as K_{1A} and K_{1B}, where the former is the member of the singlet and the second is the member of the triplet, and they are mixed and they form the observable physical states $K_1(1270)$ and $K_1(1400)$ as follows

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} sin\theta_{K_1} & cos\theta_{K_1} \\ cos\theta_{K_1} & -sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}, \tag{2}$$

where θ_{K_1} is defined as the K_1 mixing angle [17-19]. In addition to the possibility of measuring R_{K_1} through semileptonic $B \to K_1$ transitions and testing recent LHCb findings, the mixing nature of these states keeps igniting scientific curiosity.

In literature, $K_1(1270)$ and $K_1(1400)$ states and their mixing are studied widely by different approaches; however, no consensus has been reached on the value of θ_{K_1}. By using the experimental data as of 1977, Carnegie *et al.* found that $\theta_{K_1} = 33^o$ [20]. In Suzuki (1993) [17], the mixing angle θ_{K_1} was estimated via analysis of partial decay widths and masses, and $\theta_{K_1} = 33^o$ was favored. In Blundell *et al.*(1996), authors extracted θ_{K1} from the ratio of the weak decays $\tau \to \nu_\tau K_1(1270{,}1400)$ and found it as $\theta_{K_1} \simeq 45^o$ [21]. In Burkovsky and Goldman (1997), a nonrelativistic constituent quark model approach was used to obtain the constraint $35^o \le \theta_{K_1} \le 55^o$ [22]. An experimental analysis performed by CLEO collaboration suggested that $\theta_{K_1} = (69 \pm 16 \pm 19)^o$ or $\theta_{K_1} = (49 \pm 16 \pm 19)^o$ [23]. In Cheng (2003), the author analyzed $D \to K_1(1270{,}1400)\pi$ decays in Isgur-Scora-Grinstein-Wise quark model and concluded that negative values of the mixing angle were allowed and $\theta_{K_1} \simeq -58^o$ was favored [24]. In Roca *et al.* (2004), a phenomenological Lagrangian was proposed for the members of axial vector SU(3) nonet, and from the ratio of the branching ratios, the mixing angle was estimated as $30^o \le \theta_{K_1} \le 60^o$ favoring $\theta_{K_1} \simeq 45^o$ [25]. In Li and Li (2006), a nonrelativistic quark model study was performed and by comparing strong decays of $K_1(1270)$ and $K_1(1400)$ states, mixing angle was estimated as $\theta_{K_1} = \pm(59.29 \pm 2.87)^o$ [26]. In a light cone QCD sum rules analysis, Hatanaka and Yang (2008) estimated the mixing angle from the ratio of radiative B decays and favored $\theta_{K_1} = -(34 \pm 13)^o$ [27]. In Cheng (2012), the author investigated axial meson mixing through isosinglet and isotriplet mixing angles via Gell-Mann Okuba relations and obtained that $\theta_{K_1} \le 45^o$ was favored [28]. In Divotgey *et al.* (2014), authors studied explicit breaking of flavor symmetry in a relativistic effective model and obtained $|\theta_{K_1}| = (33.6 \pm 4.3)^o$ [29]. In Liu *et al.* (2014), a perturbative QCD (pQCD) analysis for $B^\pm \to \phi K_1^\pm(1270{,}1400)$ decays was performed and $\theta_{K_1} \simeq 33^o$ was favored [30]. In Zhang *et al.* (2018), authors conducted a pQCD analysis for $D^+ \to J/\psi K_1^\pm(1270{,}1400)$ decays and supported that $\theta_{K_1} \simeq 33^o$ [31]. Lastly, BESIII collaboration observed the semileptonic $B^\pm \to \overline{K}_1^0\,(1270)e^+\nu_e$ decay and reported its branching fraction for the first time, which supported the values $\theta_{K_1} = 33^o$ or $\theta_{K_1} = 57^o$, and ruled out negative possibilities. In addition to these studies concerning the axial vector mixing angle, and related to recent LHCb observation on the violation of lepton universality [10], there are several approaches to investigate the new physics beyond SM through properties and decays of $K_1(1270)$ and $K_1(1400)$ states, which used models such as Supersymmetry, Non-universal *Z'*, Leptoquarks, Two Higgs Doublet and Fourth Generation, and all of which concluded that transitions involving K_1 states are sensitive to New Physics, hence sensitive to θ_{K_1} as well [29 - 39]. As seen from this introductory summary of the status of θ_{K_1}, its value has not been determined yet, and it needs further investigations.

In this work, motivated by the findings listed above, we are revisiting K_1 states to estimate θ_{K_1} by a theoretical analysis for the very first time by using QCD Sum Rules. In this analysis, we benefit from the orthogonally of the physical states and follow the mechanism introduced by Sugiyama *et al.* (2007) [40], and Aliev *et al.* (2011) [41], to estimate the value of θ_{K_1}. We also perform a detailed Monte Carlo based statistical analysis to extract the numerical results.

This work is organized as follows: In section 2, we construct the sum rules and obtain the analytical expressions for $\tan\theta_{K_1}$ and θ_{K_1}. In section 3, we present the numerical analysis, our findings and discussions. In section 4, we present our summary and concluding remarks.

2. Theoretical Framework

In QCD, Sum Rules (QCSR) are widely and very successfully applied to study physical properties of hadrons, such as masses, decay constants, couplings or form factors (for foundations and applications, see References [42-44]). In QCDSR, the main aim is to calculate the correlation function, and for axial vector states with one Lorentz index it is written as

$$\Pi_{\mu\nu}^{H} = i\int d^4x\, e^{iqx} \langle 0 \left| T\{ j_\nu^H(x) j_\mu^{H\dagger}(0)\} \right| 0\rangle , \tag{3}$$

where T denotes the time ordered product and $j_\mu^{H\dagger}(x)$ is the interpolating current creating (the same current without dagger annihilates) the hadron tower with the quantum numbers of desired hadron H at point x. In QCDSR, this correlation function is calculated twice in the regions where hadrons are formed and also where quarks and gluons are free. While calculating the correlation function in terms of quark and gluon degrees of freedoms, operator product expansion (OPE) is used to formulate non-perturbative contributions. By equating these calculations and applying techniques to isolate the ground state of the hadron tower, such as Borel transformation and continuum subtraction, one can get the mass sum rules of hadron H (for a detailed review, please see Reference [44]). In the current problem, we will not aim to find the masses of the ground states; however, we are motivated to extract the mixing angle defined in Equation 3. Thus, we will not proceed with the conventional sum rules analysis. Instead, we will follow References [40] and [41]. In this approach we will start with the correlation function in the form

$$\Pi_{\mu\nu}^{H_1-H_2} = i\int d^4x\, e^{iqx} \langle 0 \left| T\{ j_\nu^{H_2}(x) j_\mu^{H_1\dagger}(0)\} \right| 0\rangle \tag{4}$$

where H_1 is $K_1(1270)$ and H_2 is $K_1(1400)$. Since $K_1(1270)$ and $K_1(1400)$ are mass eigenstates and they are assumed to be orthanormal, i.e. $j_\mu^{H_1}$ creates (or its dagger annihilates) only H_1, and $j_\mu^{H_2}$ does the same for only H_2, the above correlation function will naturally give zero, i.e.

$$j_\mu^{K_1(1270)} = sin\theta_{K_1} j_\mu^A + cos\theta_{K_1} j_\mu^B,$$

$$j_\mu^{K_1(1400)} = cos\theta_{K_1} j_\mu^A - sin\theta_{K_1} j_\mu^B, \tag{6}$$

where j_μ^A and j_μ^B are the interpolating currents of the orbital angular momentum eigenstates K_{1A} and K_{1B}, and one has to choose proper currents for these. Even though these states are axial vector states, in SU(3) symmetry K_{1A} couples to pure axial vector current and K_{1B} couples to pure tensor current. When SU(3) symmetry is broken, the intermingling is only at the order of their Gagenbauer moments, which is either zero or negligible. Thus, pure axial vector and tensor currents can be used in calculating the correlation function given in Equation 4 [18,19,27]. The relevant matrix elements that appear in the first step of calculation of the vanishing correlation function provided in Equation 4 are given as

$$\langle K_{1A}(\epsilon)|\bar{s}\gamma_\mu\gamma_5 d|0\rangle = i f_A m_A \epsilon_\mu^* ,$$

$$\langle K_{1B}(\epsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 d|0\rangle = -i f_B m_B (\epsilon_\mu^* p_\nu - \epsilon_\nu^* p_\mu) , \tag{7}$$

and

$$\langle K_{1A}(\epsilon)|\bar{s}\sigma_{\mu\nu}\gamma_5 d|0\rangle = -i\, a_{0,A}^{\perp} f_A m_A (\epsilon_\mu^* p_\nu - \epsilon_\nu^* p_\mu) \simeq 0 ,$$

$$\langle K_{1B}(\epsilon)|\bar{s}\gamma_\mu\gamma_5 d|0\rangle = i\, a_{0,B}^{\parallel}(1GeV) f_B m_B \epsilon_\mu^* \simeq 0 , \tag{8}$$

where shorthand notation A and B used to denote K_{1A} and K_{1B} in sub-indices, $f_{A,B}$ and $m_{A,B}$ are the decay constants and the masses of $K_{1(A,B)}$ states. In Equation 8, $a_{0,A}^{\perp}$ and $a_{0,B}^{\parallel}$ are the zeroth order Gagenbauer moments which vanish under SU(3) symmetry, and which are negligible when SU(3) symmetry is broken. This will let us define the interpolating currents as $j_\mu^A = \bar{s}\gamma_\mu\gamma_5 d$ and $j_\mu^B = \bar{s}\sigma_{\mu\nu}\gamma_5 p^\nu d$. Substituting these interpolating currents in the vanishing correlation function given in Equation 4 and playing with some algebra, one gets the relation

$$tan\theta_{K_1}\left(C_A^2 \Pi_{\mu\nu}^{AA} - C_B^2 \Pi_{\mu\nu}^{BB}\right) + \left(1 - tan^2\theta_{K_1}\right) C_A^* C_B \Pi_{\mu\nu}^{AB} = 0 + \cdots \tag{9}$$

where $C_A = (if_A m_A)^{-1}$ and $C_B = (-if_B m_B^2)^{-1}$, and we used the polarization formula $\epsilon_\mu \epsilon_\nu^* = -g_{\mu\nu} + \frac{p_\mu p_\nu}{m^2}$. In Equation 9, $\Pi_{\mu\nu}^{AA}$, $\Pi_{\mu\nu}^{BB}$ and $\Pi_{\mu\nu}^{AB}$ are the correlation functions in terms of pure states K_{1A} and K_{1B}, which appear in the vanishing correlation function, and they can be written as

$$\Pi_{\mu\nu}^{AA}(p^2) = i\int dx\, e^{ipx}\, Tr[(\gamma_\mu\gamma_5)^\dagger iS_d(-x)(\gamma_\nu\gamma_5) iS_s(x)] ,$$

$$\Pi_{\mu\nu}^{AB}(p^2) = i\int dx\, e^{ipx}\, Tr[(\gamma_\mu\gamma_5)^\dagger iS_d(-x)(\sigma_{\nu\beta}p^\beta\gamma_5) iS_s(x)] ,$$

$$\Pi^{BB}_{\mu\nu}(p^2) = i\int dx\, e^{ipx}\, Tr[\left(\sigma_{\mu\alpha}p^{\alpha}\gamma_5\right)^{\dagger} iS_d(-x)\left(\sigma_{\nu\beta}p^{\beta}\gamma_5\right) iS_s(x)]\ , \tag{10}$$

where for $q = d, s$ light quarks, the light propagator in position space up to quark condensates in the form

$$iS_q(x) = \frac{i\,\gamma.x}{2\,\pi^2 x^4} - \frac{m_q}{4\,\pi^2 x^2} - \frac{\langle q\bar{q}\rangle}{12}\left(1 - \frac{im_q}{4}\gamma.x\right) \tag{11}$$

is used in calculations. As seen from the Lorenz indices of correlation functions of pure states given in Equation 10, they have two Lorentz indices and any object carrying two Lorentz indices can be expended in terms of structures $p_\mu p_\nu$ and $g_{\mu\nu}$ as follows

$$\Pi_{\mu\nu} = \Pi' g_{\mu\nu} + \Pi p_\mu p_\nu\ , \tag{12}$$

where we omitted the superscripts A and B for simplicity, and Π' and Π are the coefficients of the Lorentz structures $p_\mu p_\nu$ and $g_{\mu\nu}$. As seen in Equation 9, any structure and its coefficient can be used to construct the sum rules. After choosing structure $p_\mu p_\nu$, and inserting the definition of the quark propagator, we proceed with Borel transformation with respect to Borel parameter M^2, apply quark hadron duality, and define the Borel transformed coefficients as

$$\overline{\Pi}^{ij}(s_0, M^2) = \frac{1}{\pi}\int_{s_m}^{s_0} ds\, e^{-s/M^2}\, Im[\Pi^{ij}], \tag{13}$$

where s_0 is the continuum threshold, and s_m is the kinematical limit. The analytical expressions of the correlation functions of the pure states are obtained as

$$\overline{\Pi}^{AA}(s_0, M^2) = \left(1 - e^{-\frac{s_0}{M^2}}\right)\left(\frac{M^2}{12\,\pi^2} - \frac{m_s\langle s\bar{s}\rangle}{3M^2}\right),$$

$$\overline{\Pi}^{AB}(s_0, M^2) = \frac{M^2}{8\pi^2}\left(1 - \frac{3}{\pi^2}\right)(M^2 - s_0)e^{-\frac{s_0}{M^2}} + \frac{m_s\langle d\bar{d}\rangle}{3},$$

$$\overline{\Pi}^{BB}(s_0, M^2) = \frac{-m_s M^2}{8\pi^2}\left(1 - e^{-\frac{s_0}{M^2}}\right) + \frac{\langle s\bar{s}\rangle - \langle d\bar{d}\rangle}{3}. \tag{14}$$

In terms of these coefficient, we get the analytical expression for the sum rules of the mixing angle of $K_1(1270)$ and $K_1(1400)$ states for the first time as follows

$$\tan\left(2\theta_{K_1}\right) = \frac{-2C_A^* C_B \overline{\Pi}^{AB}}{C_A^2\overline{\Pi}^{AA} - C_B^2\overline{\Pi}^{BB}}\,. \tag{15}$$

3. Numerical Analysis, Results and Discussions

After constructing the framework to study θ_{K_1}, we now proceed with estimating its numerical value. For this purpose, we need to determine the input parameters, such as quark masses, condansate values, decay constants and the masses of K_{1A} and K_{1B}. These input parameters are taken from literature and provided in Table 1.

$m_s = 95 \pm 5\ MeV$
$\langle d\bar{d}\rangle = -(240 \pm 10\ MeV)^3$
$\langle s\bar{s}\rangle = 0.8\ \langle d\bar{d}\rangle$
$m_A = 1.31 \pm 0.06\ GeV$
$m_B = 1.34 \pm 0.08\ GeV$
$f_A = 250 \pm 13\ MeV$
$f_B = 190 \pm 10\ MeV$

Table 1: Input parameters (all taken from Reference [44]).

As seen in the previous section, the expression of $tan\left(2\theta_{K_1}\right)$ obtained in Equation 15 does not depend on the properties of $K_1(1270)$ and $K_1(1400)$ states, but consists of two auxiliary parameters s_0 and M^2 arising from the QCDSR mechanism, and physical input parameters provided in Table 1. Physical parameters are taken from literature, however the values of auxiliary parameters s_0 and M^2 are problem dependent. The continuum threshold, s_0, was introduced in to formalism from duality of quarks and hadrons in the infinite tower contributing the calculation of correlation functions; thus, it should be greater than the square of the mass of the hadron under investigation. In conventional sum rules, it is usually chosen as $s_0 = (m_H + \delta)^2$, where δ varies between $0.3\ GeV^2$ and $0.8\ GeV^2$. On the other hand, Borel mass parameter (M^2) is used to suppress the subtraction terms, i.e. to suppress the additional terms denoted with dots appearing in Equation 5. In traditional QCDSR calculations, the obtained results usually get an uncertainty at the order of 30%, where the main source is s_0. Therefore, the values of parameters s_0 and M^2 should be chosen very carefully. Usually working regions of the QCDSR expressions are determined by the following criteria:

- The physical results obtained from QCDSR should weakly depend on s_0 and M^2,
- Working regions of s_0 and M^2 should satisfy pole dominance; i.e., the constraint on the ratio

$$\tilde{R}^{ij} = \overline{\Pi}^{ij}(s_0, M^2) \Big/ \overline{\Pi}^{ij}(\infty, M^2) > 0.5 \tag{16}$$

  should be satisfied for all $\overline{\Pi}^{ij}$ given in Equation 14.

- Within the working regions of s_0 and M^2, OPE convergence should be guaranteed; i.e., the constraint on the ratio

$$\tilde{R}^{ij}(OPE) = \overline{\Pi}^{ij,D3}(s_0, M^2) \Big/ \overline{\Pi}^{ij}(s_0, M^2) < 0.2 \tag{17}$$

should be satisfied for all $\overline{\Pi}^{ij}$ given in Equation 14, where $\overline{\Pi}^{ij,D3}$ denotes the contributions of dimension 3 terms in the correlation functions.

In order to observe the dependence of θ_{K_1} on s_0 and M^2, we plotted tan $(2\theta_{K_1})$ vs s_0 and M^2 in Figure 1, and θ_{K_1} vs s_0 and M^2 in Figure 2. It is seen from both graphs that dependence on both parameters are mild, which is not enough to provide any constraint on s_0 and M^2.

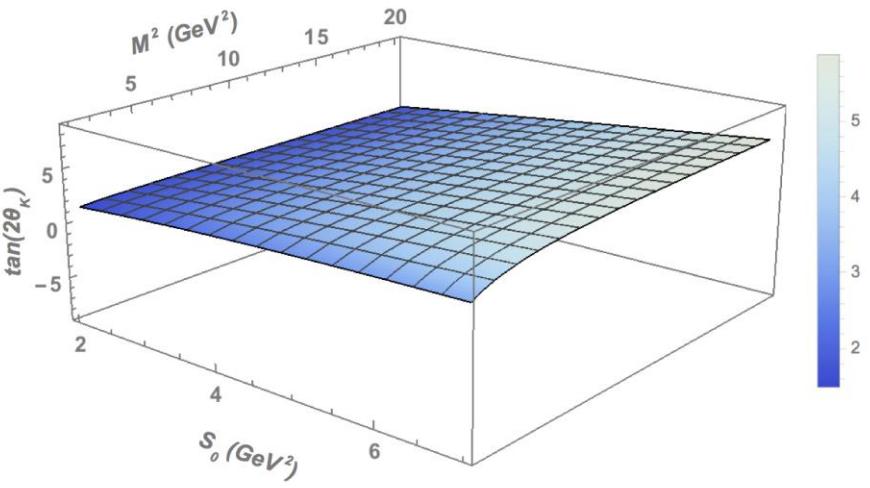

Figure 1. Dependence of $tan2\theta_{K_1}$ to s_0 and M^2 .

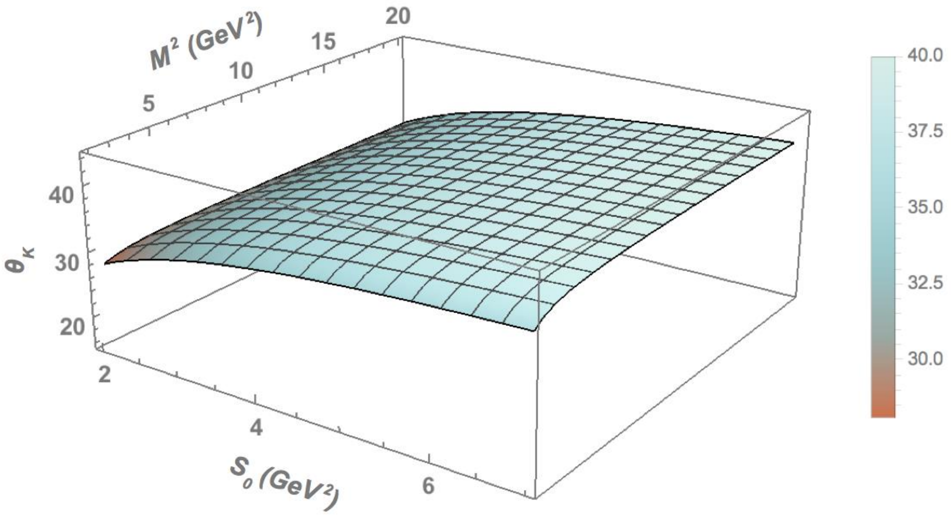

Figure 2. Dependence of θ_{K_1} (in degrees) to s_0 and M^2 .

To analyze the pole dominance and OPE convergence, we plotted $\tilde{R}^{ij} > 0.5$ and $\tilde{R}^{BB}(OPE) < 0.2$ regions in s_0 - M^2 plane for correlations functions given in Equation 14 in Figure 3, where $\tilde{R}^{AB}(OPE) < 0.2$ and $\tilde{R}^{AA}(OPE) < 0.2$ are satisfied in the whole plane; hence, they are not shown. It is seen from Figure 3 that main constraint on the working regions of continuum threshold and Borel mass comes from $\tilde{R}^{AB}$, and sets an additional condition, such as $M^2 \leq 1.05\ s_0^2 - 0.45\ GeV^2$. Since regions for $\tilde{R}^{AA}$ limit the pole dominance around $s_0 \simeq 2.5\ GeV^2$, we set the constraint $s_0 > 2.5\ GeV^2$, consistent with traditional QCDSR analysis [18,42-45].

Combining the interpretations from Figures 1, 2, 3 and 4, we set the working regions of s_0 and M^2 as follows

$$2.5\ GeV^2 < s_0 < 4.0\ GeV\text{^}2,$$
$$2.15\ GeV^2 < M^2 < 3.75\ GeV^2, \tag{18}$$

where the upper bound of s_0 is borrowed from traditional QCDSR analysis, and an additional condition

$$M^2 < 1.05\ s_0 - 0.45\ GeV^2\ , \tag{19}$$

is obtained from Figure 3, following discussions in Ref. [45].

In traditional QCDSR analysis, it is not possible to consider the constraint given in Equation 19, which gives the relation between M^2 and s_0. In addition, the uncertainties are obtained by varying parameters between their minimum and maximum values, which result in higher relative errors. In order to improve the reliability of our results, we perform a Monte Carlo (MC) based statistical analysis following Mutuk (2021), where further discussions, additional comments and a literature summary can be found [46]. Since we constructed a relation between M^2 and s_0., we did not impose a χ^2 test. For MC analyses, we generated 10^6 Gaussian distributed input values for M^2 and s_0, within the range given in Equation 18. Then we filtered the results satisfying the relation given in Equation 19. Finally we imposed the conditions $s_0^2 > m_H^2$ and $M^2 > 2.15\ GeV^2$ to filter out unphysical results. This process resulted in $n = 362476$ data points. We plotted the distributions of generated and filtered data sets of M^2 and s_0 in Figure 4. As seen from the Figure, allowed values of s_0 are shifted to the right, and allowed values of M^2 shifted to the left, and remaining data of n points could produce statistically reliable results.

Finally, we generated n Gaussian distributed values for input parameters provided in Table 1, and by inserting this data in analytical expression given in Equation 15 and computing it n times, we get the resulting set of values for $\tan(2\theta_{K_1})$. We plotted these values and their corresponding angles in the first quadrant in histograms in Figure 5. It is seen from Figure 5 that both distributions can be modeled as a dimidiated Gaussian, and where $\tan(2\theta_{K_1})$ has a longer tail in the right side, θ_{K_1} has a longer tail in the left side of the distributions. Even though the distributions of the input sets are almost Gaussian, the resulting distributions can be approximated as a dimidiated Gaussians, i.e. two Gaussians with different standard deviations around the same mean. The histograms provided in Figure 5 corresponds to numerical results

$$\tan(2\theta_{K_1}) = 6.04^{+1.49}_{-1.34}\ , \tag{20}$$

and

$$\theta_{K_1} = 39.91^{+1.05\ o}_{-1.42}\ , \tag{21}$$

where in obtaining asymmetric errors, we followed the recipe given in Barlow (2002) for dimidiated Gaussian distributions [47].

The results obtained in this study are the first ever QCDSR estimation of θ_{K_1}, and despite the dissension on the value of θ_{K_1} in literature, our results are in good agreement with references favoring positive values [17,20-23,25,26,28-30], including experimental determination by BESIII[31].

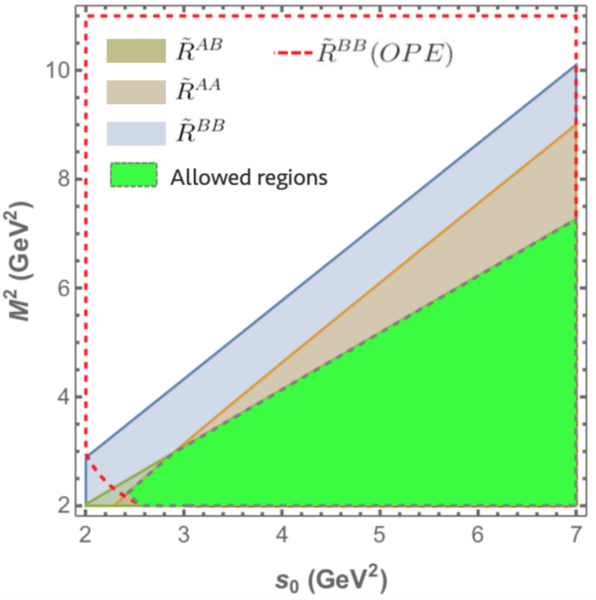

Figure 3. Regions satisfying pole dominance, i.e., $\tilde{R}^{ij} > 0.5$ and OPE convergence for $\tilde{R}^{BB}(OPE) < 0.2$ in s_0 - M^2 plane. $\tilde{R}^{ij}$ are described by filling colors on plot. The boundary of $\tilde{R}^{BB}(OPE) < 0.2$ is given by dashed red curve. Allowed values of s_0 and M^2 are in green.

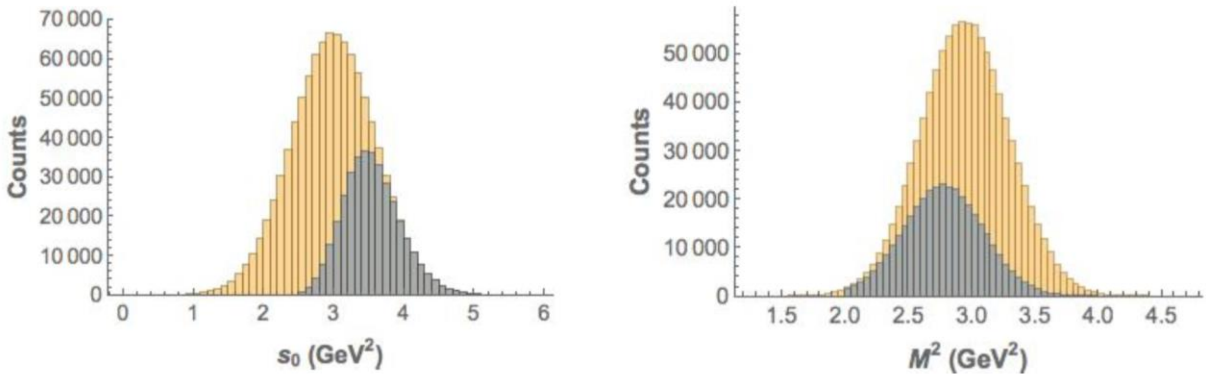

Figure 4. The histogram of Gaussian generated (yellow) and filtered (gray) s_0 (left) and M^2 (right) values.

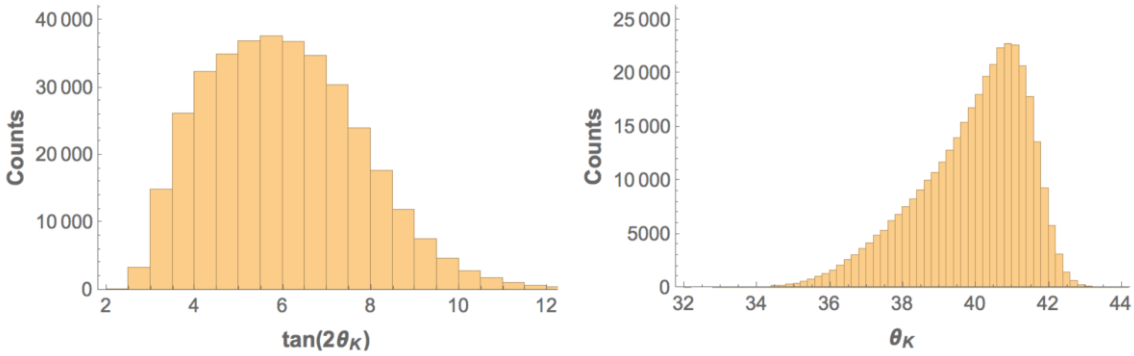

Figure 5. The histogram of $tan2\theta_{K_1}$ (left) and θ_{K_1} in degrees (right) from 362476 matches.

4. Conclusions

In the present work, we employed a theoretical QCDSR based investigation to estimate the mixing angle between axial vector *K₁(1270)* and *K₁(1400)* mesons. By using interpolating currents of pure K_{1A} and K_{1B} states and benefiting from the orthogonality of the physical states, we calculated the analytical expression for $\tan(2\theta_{K_1})$. We carefully determined the working regions of QCD Sum Rules. We generated 10^6 Gaussian distributed data for continuum threshold s_0, and Borel mass M^2. We filtered out the data which do not belong to working regions and end up with $n = 362476$ data points. For the remaining input parameters, we generated n Gaussian distributed data, and calculated $\tan(2\theta_{K_1})$ for n times. We finally plotted $\tan(2\theta_{K_1})$ and θ_{K_1} in histograms and calculated the means and asymmetric errors by assuming that the distributions are dimidiated Gaussian. We found the following:

$$\tan(2\theta_{K_1}) = 6.04^{+1.49}_{-1.34} \quad \text{and} \quad \theta_{K_1} = 39.91^{+1.05}_{-1.42}{}^{o} \, . \tag{22}$$

This is the first theoretical estimation of the mixing angle via QCDSR. Our findings are favoring positive values of the mixing angle, and they are in good agreement with the major findings in the literature. This result can be used in estimating the lepton universality ratio R_{K_1} for axial vector *K₁(1270)* and *K₁(1400)* mesons, and it can be tested at high energy colliders if R_{K_1} will be measured. Finally, our result can be improved by calculating higher order non-perturbative contributions.

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References

[1] Oerter, R. (2006). The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics Penguin Group. p. 2. ISBN 978-0-13-236678-6.

[2] Yang, C. N. and Mills, R. (1954). "Conservation of Isotopic Spin and Isotopic Gauge Invariance". Physical Review. 96 (1).

[3] Glashow, S.L. (1961). "Partial-symmetries of weak interactions". Nuclear Physics 22 (4), 579–588.

[4] Weinberg, S. (1967). "A Model of Leptons". Physical Review Letters 19 (21), 1264–1266.

[5] Salam, A. (1968). Svartholm, N. (ed.). Elementary Particle Physics: Relativistic Groups and Analyticity. Eighth Nobel Symposium. Stockholm: Almquvist and Wiksell. p. 367.

[6] Englert, F. and Brout, R. (1964). "Broken Symmetry and the Mass of Gauge Vector Mesons". Physical Review Letters. 13 (9), 321–323.

[7] Higgs, P.W. (1964). "Broken Symmetries and the Masses of Gauge Bosons". Physical Review Letters. 13 (16), 508–509.

[8] CMS collaboration (2012). "Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC". Physics Letters B 716 (1), 30–61. arXiv:1207.7235.

[9] ATLAS collaboration (2012). "Observation of a New Particle in the Search for the Standard Model Higgs Boson with the ATLAS Detector at the LHC". Physics Letters B 716 (1), 1–29. arXiv:1207.7214.

[10] Aaij, R. et al., LHCb Collaboration (2021). "Test of lepton universality in beauty-quark decays", LHCb-PAPER-2021-004, CERN-EP-2021-042 e-Print: arXiv:2103.11769 [hep-ex].

[11] Aaij, R. et al., LHCb Collaboration , (2014). "Differential branching fractions and isospin asymmetries of B → K(*)μ +μ − decays", JHEP 06 , 133, arXiv:1403.8044[hep-ex].

[12] Aaij, R. et al., LHCb Collaboration, (2015). "Angular analysis and differential branching fraction of the decay B0 s → φμ+μ −", JHEP 09 (2015) 179, [1506.08777].

[13] Hiller, G. and Kruger, F. (2004). "More model-independent analysis of b → s processes", Phys. Rev. D69, 074020, arXiv:0310219[hep-ex].

[14] Bobeth, C., Hiller, G. and Piranishvili, G. (2007). "Angular distributions of B¯ → K ¯ll decays", JHEP 07, 040, arXiv:0709.4174[hep-ex].

[15] Bordone, M., Isidori, G. and Pattori, A. (2016). "On the Standard Model predictions for RK and RK*", Eur. Phys. J. C76 440.

[16] Zyla, P.A. et al. (Particle Data Group). (2020). "The Review of Particle Physics", Prog. Theor. Exp. Phys. 2020, 083C01.

[17] Suzuki, M. (1993), "Strange axial-vector mesons", Phys. Rev. D 47, 1252.

[18] Dağ, H. (2010). "Investigating the Semileptonic B to K1(120,1400) Decays in QCD Sum Rules", PhD Thesis, Middle East Technical University, Ankara, Turkey, 83p.

[19] Dağ, H. et al. (2011). "The Semileptonic B to K1(1270,1400) Decays in QCD Sum Rules", J. Phys. G38, 015002.

[20] Carnegie, R.K. et al. (1977). "Q1(1290) and Q2 (1400) decay rates and their SU(3) implications", Physics Letters B Volume 68, Issue 3, Pages 287-291.

[21] Blundell, H.G. et al. (1996), "Properties of the Strange Axial Mesons in the Relativized Quark Model", Phys. Rev. D 53, 3712-3722.

[22] L. Burakovsky and T. Goldman, "Constraint on axial-vector meson mixing angle from the nonrelativistic constituent quark model", Phys. Rev. D 56, R1368(R)

[23] Asner, D., et al. (2000). "Resonance structure of τ-→K-π+π-ντ decays", Phys. Rev. D, 62(7).

[24] Cheng, H-Y. (2003), "Hadronic Charmed Meson Decays Involving Axial Vector Mesons", Phys. Rev. D 67 (2003) 094007.

[25] Roca, L. et al. (2004). "Decay of axial-vector mesons into VP and Pγ", Phys. Rev. D 70, 094006, arXiv:0306188[hep-ph].

[26] Li, D-M. and Li, Z. (2006). "Strange axial-vector mesons mixing angle", Eur. Phys. J. A 28, 369-373, arXiv: 0606297[hep-ph].

[27] Hatanaka, H. and Yang, K-C. (2008). "B→K1γ Decays in the Light-Cone QCD Sum Rules", Phys. Rev. D77, 094023, 2008; Erratum-ibid.D78:059902,2008, arXiv:0804.3198v4 [hep-ph].

[28] Cheng, H-Y. (2012). "Revisiting Axial-Vector Meson Mixing", Phys. Lett. B 707, 116-120, e-Print: 1110.2249 [hep-ph].

[29] Divotgey, F. et al. (2014). "Phenomenology of axial-vector and pseudovector mesons: decays and mixing in the kaonic sector", Eur. Phys. J. A 49, 135, arXiv:1306.1193.

[30] Liu, X. et al. (2014). "Penguin-dominated B→ϕK1(1270) and ϕK1(1400) decays in the perturbative QCD approach", Phys. Rev. D 90, 094019, arXiv:1404.2089v2 [hep-ph].

[31] Zhang, Z-Q. et al. (2018). "Study of the K1(1270)−K1(1400) mixing in the decays B→J/ΨK1(1270),J/ΨK1(1400)", Eur. Phys. J. C 78, 219, arXiv:1705.00524 [hep-ph].

[32] Bashiry, V. and Azizi, K. (2010). "Forward-backward asymmetry, branching ratio and rate difference between electron and muon channels of B → K 1(K*)ℓ+ℓ− transition in supersymmetric models", JHEP 1001, 033, arXiv:0903.1505 [hep-ph].

[33] Ahmed, I. et al. (2008). "Exclusive B→K1ℓ+ℓ- decay in model with single universal extra dimension", Eur. Phys. J. C 54, 591-599, arXiv:0802.0740 [hep-ph].

[34] Ahmed, A. et al. (2011). K1(1270)−K1(1400) mixing and the fourth generation standard model effects in B→K1l+l− decays, Phys. Rev. D 84, 033010, arXiv:1105.3887 [hep-ph].

[35] Li, Y. et al. (2011). "B→K1ℓ+ℓ− decays in a family non-universal Z′ model", EPJC 71, 1775, arXiv:1107.0630 [hep-ph].

[36] Ahmed, N. et al. (2015). "Analysis of forward–backward and lepton polarization asymmetries in B → K1ℓ+ℓ− decays in the two-Higgs-doublet model", PTEP 2015, 1, 113B06, arXiv:1509.08113 [hep-ph].

[37] Munir, F. et al. (2016). "Polarized forward-backward asymmetries of lepton pair in B→K1ℓ+ℓ− decay in the presence of New physics", PTEP 2016 1, 013B02, arXiv:1511.07075 [hep-ph].

[38] Huang, Z-R. et al. (2019). "Testing Leptoquark and Z′ Models via B→K1(1270,1400)μ+μ− Decays", Phys. Rev. D 100, 055038, arXiv:1812.03491 [hep-ph].

[39] Bhatta, A. and Mohanta, R. (2020). "Implications of new physics in B→K1μ+μ− decay processes", arXiv:2011.05820 [hep-ph].

[40] Sugiyama, J. et al. (2007). "Mixings of 4-quark components in light non-singlet scalar mesons in QCD sum rules", Phys. Rev. D76, 114010, arXiv:0707.2533 [hep-ph].

[41] Aliev, T. et al. (2011). "Mixing Angle of Hadrons in QCD: A New View", Phys. Rev. D 83, 016008, arXiv:1007.0814 [hep-ph].

[42] Belyaev, V.M. and Ioffe, .L. (1982). "Determination of Baryon and Baryonic Resonance Masses from QCD Sum Rules". 1. Nonstrange Baryons, Sov. Phys. JETP 56, 493-50.

[43] Shifman, M.A. et al. (1979). "QCD and Resonance Physics. Theoretical Foundations", Nucl. Phys. B 147 (1979) 385-447.

[44] Colangelo, P. and Khodjamirian, A., (1995). "QCD Sum Rules, a Modern Perspective", printed in ”At the Frontier of Particle Physics: Handbook of QCD” ed. by M. Shifman (World Scientific, Singapore, 2001), V. 3.

[45] Türkan, A. and Dağ. H. (2019).” Exploratory study of X_{c1} (4140) and like states in QCD sum rules”, Nucl. Phys. A 985, 38-65.

[46] Mutuk, H. (2021). "Monte-Carlo based QCD sum rules analysis of X0(2900) and X1(2900)", J. Phys. G 48 , 5, 055007, e-Print: 2009.02492 [hep-ph].

[47] Barlow, R. J. (2002). “Systematic Errors: Facts and Fictions” in Proc. Durham conference on Advanced Statistical Techniques in Particle Physics, M. R. Whalley and L. Lyons (Eds). IPPP/02/39. 2002.