

# Local Adaptive Phase Correction Algorithm For 3-D Profilometry Systems Based on Phase Shifting Method

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# Abstract

The method of reflecting sinusoidal phase-shifted patterns to the surface, based on the demodulation technique of phase information, has been a popular method to obtain 3-D surface depth using 2-D images. The phase information that is extracted with this method is wrapped; so it must be unwrapped. Even though the phase information is unwrapped, there will be some errors because of the possibility of non-sinusoidal characteristics of phase patterns, surface discontinuities, low sample rates, and technical handicaps (poor calibration and hardware malfunctions, and so on). To deal with these errors resulting from the phase unwrapping process, there are some computationally expensive and complex methods that have been presented. In this paper, a fast and low complex local adaptive phase correction algorithm based on the four-step phase shifting method is implemented. The method is firstly validated by using synthetic data. After the validation process, an optic test system is realized, and a few experiments are performed by using physical real data. For the optical system used to physically acquire the data, a lookup table-based calibration technique has also been developed to obtain accurate surface phase information. The performance of the method is evaluated with simulation results and real data, and visually compared to popular unwrapping methods.

Keywords: 3-D reconstruction, fringe patterns, phase shifting, phase unwrapping.

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### 1. Introduction

It is very convenient that using non-contact optical methods in determining 3-D surface profiles is common in Computer Aided Manufacturing (CAM) systems. The most popular method of these optical techniques is projecting the beam patterns (fringe patterns) onto the surface to get 3-D information. This method is also known as shadow moire method [1]. Patterns keep the depth information belonging to the 3-D surface as phase information which is modulated. Therefore the fringe patterns also called as phase patterns.

There are some methods for the extraction of the demodulated phase information from the modulated phase patterns. Fourier fringe analysis technique, direct phase demodulation, and phase shifting method are some examples for these analysis methods [2-4]. If the surface to be reconstructed is not dynamic, phase shifting technique is practically convenient. The method does not require complex computations. In this method, at least three phase patterns, which have equal phase intervals, are projected onto the surface so that different phase images can be obtained.

The extracted phase information is wrapped because the sinusoidal function is subjected to the arctangent function (as expressed in Equation 1). Therefore, there are discontinuities in the phase function. Typically the unwrapping process intends to transform the wrapped phase function into the continuous form. There is a basic method for the unwrapping process; however, as some unwanted issues such as surface discontinuities, low SNR, low sample rates or non-ideal sinusoidal fringe patterns may be encountered with the use of this method, the unwrapped phase will have some errors. Especially these errors are static or propagative. To overcome this problem, a number of phase correction algorithms have been implemented. Zhao and others, developed a method for the spatial light modulators which has the advantages of high-precision pixel-wise phase correction against environmental disturbance. This method relies on gamma correction and shape aberration correction, using manufacturer's gamma lookup table [5]. Thang et al. propose a method based on local 1-D phase correction by using multi-frequency phase heterodyne. The method determines whether a phase map needs to be corrected by calculating the deviation between the predicted and actual phase according to the slope of the projected standard fringe pattern and has the best effect to eliminate jump like phase errors [6]. Zheng et al. propose a fast self-correction algorithm to reduce the nonlinear phase error by using average intensity value of the captured fringe images which have nonlinear response parameters and also use multi-frequency heterodyne algorithm [7].

Song et al. propose three wavelength grating method which uses the phase information of three different wavelengths for phase unwrapping and phase correction superior than the traditional multi-frequency heterodyne method [8]. Zhang et al. address multipath problem in phase shifting profilometry applications [9]. Feng et al., present a study for phase similarity based on light field reconstruction method. Their method also uses multi-frequency heterodyne algorithm for unwrapping the phase data. Different from other studies, this light field imaging method, which is an emerging method for 3-D reconstruction studies, needs a calibration step addition to fringe projection step [10]. In addition to new analyses,

there are also studies where algorithms that will produce good results in noisy conditions are developed. Zhao et al. developed a simple and robust unwrapping algorithm, using the transport of intensity equation (TIE) and fast cosine transform. They report that their algorithm produces satisfactory unwrapping results even in a notably noise condition [11].

In fact, if there were no problems in surface phase functions mentioned above, it would be enough to use the basic unwrapping method and would need no phase unwrapping algorithms. Other processes to obtain 3-D height of the object surface after phase unwrapping process are to eliminate the slope and to use the appropriate calibration function for phase-to-height relation.

Except for Zhao's work [11], other methods that are not mentioned here, besides including quite hardware development studies requiring a calibration phase, treat the phase data in 1-D. In this study, a new Four-Step Phase Shifting Method with phase error correction algorithm is developed for optical 3-D measuring. Here, 1-D unwrapping process is applied as 2-D for phase unwrapping in the algorithm. A 2-D adaptive correction algorithm is also developed to cancel phase noises. Besides, a lookup table-based calibration algorithm is implemented to obtain accurate sinusoidal phase patterns. This calibration algorithm can be used when working with the real data, i.e. when using a physical optical system. Because of the distortions of the optical systems, especially gamma function of the optic device and surface characteristics, gray-level differences may occur between the phase patterns created on the computer and the phase patterns projected onto the surface. These distorting effects are observed as unwanted slopes and undulations on the object surface. For optic test setup, a DLP light commander module for projecting fringe patterns onto the surface and a high resolution CCD/CMOS camera for capturing images from the surface are used. After getting the accurate phase patterns, the desired phase unwrapping algorithms is then implemented and described in the sections below.

#### 2. 3-D Reconstruction Technique Based On Phase Shifting Method

In a typical fringe projection system (Figure 1), a computer generates fringe patterns of vertical sinusoidal stripes and sends them to a projector, called a light modulator, which will reflect them onto objects. As the result of Shadow Moire principle [1], the reflected fringe image patterns are modulated and contain surface 3-D information as phase values. Then reflected images are captured by a video camera. 3-D information is determined by using the triangulation technique according to the Shadow Moire principle. As the fringe patterns are sinusoidal, each vertical fringe stripe consists of phase information of the objects.



Figure 1.Real-time 3-D imaging system with fringe analysis.

Fringe patterns mentioned above are in sinusoidal form and can be expressed as [12]:

$$I(x, y) = a(x, y) + b(x, y) \cos(2\pi f_0 x + \emptyset(x, y))$$
(1)

Where a(x, y) is the background illumination, b(x, y) is the amplitude of the fringe phase modulation,  $f_0$  is the spatial carrier frequency, and  $\emptyset(x, y)$  is the phase value of the fringes in each value of (x, y) position. In order to calculate  $\emptyset(x, y)$ , there are several techniques mentioned in the introduction section, and in this study four-step phase shifting technique is used.

#### 2.1. Four-step Phase Shifting Method

The four-step phase shifting method have some advantages, such as high measurement speed, low computational complexity, cancellation of background illumination, detail accuracy, and ease of automation. In this technique, four independent equations is used to eliminate a(x, y) and b(x, y) [1]. Practically this is implemented as using  $\pi/2$  phase shifted images and expressed mathematically as below:

$$I_{1}(x, y) = a(x, y) + b(x, y) \cos(2\pi f_{0}x + \phi(x, y))$$

$$I_{2}(x, y) = a(x, y) + b(x, y) \cos(2\pi f_{0}x + \phi(x, y) + \frac{\pi}{2})$$

$$I_{3}(x, y) = a(x, y) + b(x, y) \cos(2\pi f_{0}x + \phi(x, y) + \pi)$$

$$I_{4}(x, y) = a(x, y) + b(x, y) \cos(2\pi f_{0}x + \phi(x, y) + \frac{3\pi}{2})$$
(2)

By using Equation 2, following expression can be extracted for calculating the phase value  $\phi(x, y)$ :

$$\Psi(x,y) = tan^{-1} \left(\frac{I_4 - I_2}{I_1 - I_3}\right) = 2\pi f_0 x + \phi(x,y)$$
(3)
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where  $\Psi(x, y)$  is the wrapped phase and consists of  $2\pi$  jumps. Spatial carrier frequency  $f_0$  causes a tilt in the reconstructed image, and it must be eliminated. That is achieved with subtraction of the unwrapped reference plane from the unwrapped phase values of  $\Psi(x, y)$ , which is subjected to desired unwrapping algorithm mentioned in following section.

#### 2.2. Local Adaptive Phase Error Correction Algorithm

In Equation 3, wrapped phase map values range from -  $\pi$  to  $\pi$  because of the nature of the arctan function. The classical approach for phase unwrapping problem is to add or subtract  $2\pi$  in the phase map. The main reason behind this is to make the result of unwrapping process as the same as the original true phase value  $\phi(x, y)$ .

For 1-D case, unwrapping process can be expressed mathematically as [13]:

$$U[\Psi(n)] = \Psi(n) + 2\pi k \tag{4}$$

where k is an integer and  $U[\Psi(n)]$  is the unwrapped phase to be calculated. Unwrapping process basically follows the steps below:

- 1. Start with the second sample of  $\Psi(n)$ .
- 2. Calculate the difference,  $\Delta \Psi(n) = \Psi(n+1) \Psi(n)$ .
- 3. If  $\Delta \Psi(n) > \pi$ , subtract  $2\pi$  from this sample and from all the following samples.
- 4. If  $\Delta \Psi(n) < -\pi$ , add  $2\pi$  to this sample and to all the following samples.
- 5. Repeat these 1 to 4 steps for all samples of  $\Psi(n)$ .

In fact, if there were no corrupt data in the wrapped phase map, 1-D unwrapping technique could be applied for columns and rows of the wrapped phase value, and so it could be easy to obtain unwrapped phase value  $U[\Psi(n)]$  as the same with  $\emptyset(x, y)$ . Since the ideal cases cannot be supplied, there will be error propagations in result of the classical unwrapping process. Therefore, for 2-D case, several algorithms which use binary stripe [14] and color-coded stripes [15] have been proposed for the unwrapping problem. These algorithms can be classified into two major groups as local and global unwrapping algorithms [16]. In these algorithms, accuracy is well-proportioned with complexity; however, execution time increases.

In this study, due to its low complexity of 1-D unwrapping method, 2-D phase unwrapping method is implemented by applying 1-D processes both to columns and rows of the wrapped phase map. As the resulting unwrapped phase map may have errors which cause artifacts in the reconstructed image, they are eliminated with the developed two-step recursive algorithm in this study. The algorithm tries to find the artifacts and correct them by using adjacent uncorrupted phase pixel values. This process is applied for whole pixel values of the wrapped phase image respectively. This approach has very low computational complexity and runs very fast; the pseudo code of the algorithm is given below.

- Apply 1-D unwrapping process to each columns of the wrapped phase map.
- Assign each row vector to a blank row of an empty matrix.
- For each pixel in the row of matrix,
  - o Calculate the differences between gray value of consecutive pixel

$$D_i = p(x, y) - p(x + i, y)$$
 (5)

If the absolute difference is greater than the threshold level  $(|D_i| > \varepsilon)$  which is determined by the user,

• assign the gray level value of the corresponding pixel of *i*th, to a variable of  $m_1$ ,

 $m_1 = p(x+i, y)(6)$ 

- start a new seek along a user defined 1D window length (*m*)
- calculate the difference between the gray levels of the pixels in the window (W) and  $m_1$
- If the absolute difference is greater than the threshold level, which may be different from the above, continue seeking along the window length

 $Dw_k = m_1 - W(\hat{\imath}), \qquad \hat{\imath} = 0..m, k = 1..m$  (7)

• If the absolute difference is lower than the threshold level, assign the gray level value of the corresponding pixel of kth, to a variable of  $m_2$ 

$$if |Dw_k| < \varepsilon$$
 ,  $m_2 = W(\hat{\imath})$  (8)

• change the gray levels of the pixels among i and k with equal intervals of  $m_1$  and  $m_2$ 

 $p(x+c,y) = \left[\frac{m_2 - m_1}{k-i}\right], \ c = i, \dots, k, | \ c, i, k \in \mathbb{Z}$  (9)

- o If the absolute difference is lower than the threshold level which is determined by the user,
  - continue the seeking process along the row matrix length.
- Repeat the same steps for each column of the matrix.

The pseudo code reveals that the algorithm has a threshold level and a window length parameter for operation. These values can be set by the user and changed for the application to get optimum results. Because of changing the row data along the algorithm steps, it works clearly fast. Since the scanning process is performed according to the difference between gray-level value and the threshold level, the algorithm works adaptively.

Figure 2 shows the corrective effect of the algorithm on the wrapped phase map. Reconstructed 3-D image obtained from 1-D unwrapping process is given in Figure 2a. As can be seen from the figure, there are phase errors needed to be eliminated. After applying proposed adaptive algorithm, corrected unwrapping result is obtained as shown in Figure 2c. Figure 2b and Figure 2d indicate selected row data from 3-D images on left.



Figure 2. Results obtained with and without the proposed algorithm. a-b) 3-D image and selected row data of it obtained without proposed method. c-d) 3-D image and selected row data of it obtained with proposed method.

# 3. System Setup And Calibration

In order to obtain a 3-D surface profile of the physical object models by applying the phase shift method, a system consisting of a projector, a computer and a camera has been established.

#### 3.1. System Setup for Acquiring Real Data

The system setup, shown in Figure 3, includes a light commander (DLP), an industrial camera and two laptops. Test system is set to obtain 3-D information of a white object which is placed in a 30 x 30 cm view area and 50 cm far from the system plane. Resolution of the camera is 5MP, it has noise cancellation ability, and image capture speed is 120 fps. The angle between the camera and projector is adjustable with a special platform. The generated fringe patterns are projected via a laptop, and the projected images are recorded via another one.



Figure 3.System setup.

There are corruptions in the images captured by the optical test system because of the optical reasons, light effects, and the surface characteristics of the real object. In Figure 4, the light profile captured from the surface and reconstructed 3-D surface using this profile is shown. It is significant that an unwanted slope occurs on the reconstructed 3-D surface, so the calibration process described in the following section is applied to eliminate such type of distortions.



**Figure 4.** An example of light disturbances: a) Fringe image, (b) Cross-sectional light distribution profile on the plane, (c) Reconstructed 3-D object.

#### 3.2. Lookup Table-Based Calibration Algorithm

In order to reconstruct the 3-D object accurately and also to achieve high performance from any phase unwrapping algorithm, it is very important to obtain the sinusoidal fringe patterns properly, so the system needs to be calibrated. In

this study, both real and synthetic data are used. While the real data is obtained through the established system, the synthetic data is obtained by using the 3-D CAD program. For the synthetic data, there is no need for system calibration because the fringe patterns can be accurately obtained. Since there may be obvious differences between the sinusoidal fringe patterns captured on the plane and generated from the light modulator, a lookup table-based calibration algorithm is proposed based on the adjustment of gray level values to make the fringe pattern on the plane ideally sinusoidal [17]. Figure 5 shows the block diagram of the lookup table-based calibration algorithm.



Figure 5. Lookup table-based calibration algorithm: a) Creating the LUT, b) Using the LUT for phase image projection

Although the camera used in the setup meets the focusing requirement in the specified range, lens distortions affect the system as well as in all camera systems. The MATLAB Camera Calibration Toolbox [18] is used to eliminate these lens distortions, which usually occur at the edges of the lens rather than the center. The calibration process is completed by projecting the chessboard pattern on the plane and obtaining the distortion pattern of the lens according to the reference pattern. After determining the distortion model for the lens, this distortion model is applied inversely to obtain a corrected

image. The light Commander Module can produce different calibration patterns, so that the calibration procedure may be performed easily.

Since the reference object used in the system is in the center of the image plane and is small enough to be exposed to less camera distortions, projection-induced distortions emerge as a more important problem than camera-induced distortions. Un-calibrated projector-induced distortion effects appear as an undesirable tilt and undulations on reconstructed objects as shown in Figure 4. Therefore, in addition to the camera calibration, the projection system (Light Commander) must also be calibrated.

Projector-related limitations are generally related to the fact that the reference pattern gray level is practically not achieved in the 0-255 range uniformly. This results in data loss and fluctuations on objects due to rounding the same value in a given range of levels in 3-D reconstruction.

The system is capable of linear operation in the gray-level sensitivity range from 50 to 160. The first step to eliminate this problem is to set the gray-level values of the patterns to this gray-level range. The system is also points out that the slope in the light profile is caused by the lens of the projector optics. In order to solve these problems, a distortion function made by the projector lens for each pixel should be determined. For this purpose, a greyscale test image sequence is generated, each of which has the same values between 0 and 255. These images are then sequentially projected onto the object-free surface. A MATLAB-based algorithm which calculates the lookup table value to obtain the required reference image is developed. In this algorithm, the values required for each pixel and the values generated by the projection system are calculated by matching with a lookup table-based method, and this is applied to the reference pattern. Thus, the desired image is obtained, and unwanted tilts (or slopes) in the light profile can be largely eliminated. The flow chart of the algorithm is given in Figure 6.



Figure 6. Algorithm flowchart.

The reference pattern obtained by using the proposed algorithm and the corrected phase pattern are shown in Figure 7. This algorithm is needed to perform the calibration process before the first use of the system.



**Figure 7.** Result of proposed calibration algorithm: (a) Reference phase pattern, (b) Corrected phase pattern resulted from proposed algorithm to obtain reference pattern of (a).

# 4. Simulation Results

In the simulation studies, the proposed algorithm are tested with synthetic data containing phase images of 3-D object models. Synthetic object model (tooth model) samples are created in the Mudbox (trial version) program on the computer, and these models are transferred to the 3D Studio Max (trial version) scene to simulate the optical system setup. All the images used in this setup have 1024x768 pixels resolution. In order to obtain phase images, phase shifted fringe patterns are produced in MATLAB and then transferred to 3D Studio Max scene. By determining the angle between camera and light modulator, fringe patterns are projected onto the 3-D object models. All of four phase shifted fringe patterns (0, 90, 180 and 270 degree) are projected successively. The camera in the virtual optic setup captures modulated fringe patterns which are reflected from 3-D object model, then wrapped phase image is obtained by using four-step phase shifting algorithm technique. This wrapped phase image is unwrapped using the proposed method, and 3-D data is obtained. Figure 8 and Figure 9 show the original 3-D object models, the wrapped phase images, and the reconstruction results obtained by proposed method, respectively.

For the reconstruction of the image, 1-D phase unwrapping process is applied to the columns of the wrapped phase image, and the scaling factor [19] is used in order to transform the unwrapped phase values into the 3-D data (height) values. The resulting 3-D data consist of a tilt because of the carrier frequency  $f_0$ , which is mentioned in Equation 3 and needs to be eliminated to obtain tilt-free 3-D data. As shown in Figure 10, this is done by subtracting the unwrapped phase image of the reference plane from the unwrapped phase image of object.



**Figure 8.** Four step phase shifting result for proposed method: (a) Original synthetic 3-D object model-1, b) Wrapped phase image, (c) Phase unwrapped and tilt removed result, (d) 3-D reconstruction result.



Figure 9. Four step phase shifting result for proposed method: (a) Original synthetic 3-D object model-2, b) Wrapped phase image, (c) Phase unwrapped and tilt removed result, (d) 3-D reconstruction result.



**Figure 10.** Tilt removing from the 3-D data: (a) Resulting image of 1-D unwrapping process to wrapped phase image, (b) Resulting image of 1-D unwrapping process to reference plane, (c) Unwrapped object with tilt removed.

In order to compare the performance of the proposed method with known methods, comparisons are made in this study with both synthetic and real data. In the tests, to determine the system angle and the number of fringes to be used, some analyses are made. Since using lower fringe numbers results in higher sensitivity and less edge noise, the fringe number 8 is chosen for all tests. The selection of the projection angle is another important issue; and at higher angle values, shadows are formed in the fringe image; and therefore, a lot of important phase information is lost. At low angles, it is observed that notches are formed on the obtained surface but can be removed with a smoothing filter. An angle of 5 to 10 degrees determined to be the right choice for the system angle for best performance.

For comparison, phase test data set consisting of 4 synthetic and 2 real data are processed with classical unwrapping (Itoh method [20]), Goldstein unwrapping method [21], Zhao's method [11], and the proposed method, and the results are presented in Figure 11. In the test process, the data and other operations are the same for all methods, but there is a difference only in terms of the unwrapping method.

Method1 (Itoh)	Method 2 (Goldstein)	Method 3 (Zhao)	Method 4 (Proposed)

Figure 11. Comparison of the proposed method with the methods of Itoh, Goldstein, and Zhao. The first 4 rows show the results for the synthetic data, and the last two rows show the results for the real data. From left to right, the first column shows the results obtained by Itoh's method, the second column refers to Goldstein's and the third column to Zhao's methods, respectively, while the last column reveals the results obtained by the proposed method.

The simulations in the study are done with a laptop of Pentium Core i5 3 GHz processor, 8.0 GB RAM, and the MATLAB version is 2019b. Except for the Goldstein algorithm, other algorithms obtain the results under 1 second, while the Goldstein algorithm has an average of 10 seconds to obtain the result. The Quality Guided unwrapping method [22] is also used in the study; however, this method needs a user input for the correct phase information, and the unwrapping process takes quite a long time (over 6 minutes). Therefore, it is not included in the comparison results.

When the results are compared visually, it is seen that all algorithms produce the same correct result for real data. While obtaining the phase image of the real data, the lookup table-based calibration algorithm developed in this study is used. Since the fringe data is obtained properly and relatively simple objects are used, it seems normal to achieve this success. However, when the results obtained by using the phase images of complex artifacts, which are taken as properly calibrated, are compared, it is seen that the proposed method achieves very good results compared to the other 3 methods. Window size is selected as 10% of image size, and threshold value is selected as 20% of local gray (phase) value for all the phase images.

## 5. CONCLUSION

A new local adaptive phase correction algorithm based on phase shifting method for 3-D reconstruction applications is presented in this study. While the method developed for the phase unwrapping process in this study is applied as modifications of the results of the basic phase unwrapping technique, other methods such as Goldstein, Quality Guided, and Least Squares [23] make some changes on the phase data before applying the unwrapping process.

For this reason, Itoh and Zhao's methods, which have similar approaches, seem to be more important in terms of comparisons. Although the fastest method is Itoh's method, robustness of this method is notably low. Therefore, there is not much difference between Zhao's and Goldstein's methods in terms of obtaining reliable results, yet the Goldstein method has the disadvantage of long processing time. The proposed method is both fast like the Itoh's and Zhao's methods and more reliable than the other methods compared. For reconstructed 3-D data,  $Z_{min}$  is -0,5077, and  $Z_{max}$  is 8,0483, and the difference between two consecutive points is 0,0020. This means that 8,54 mm object can be reconstructed with 2 µm sensitivity in the physical setup.

In recent years, superior results have been obtained thanks to the technical developments in 3-D profilometry systems. On the other hand, phase shift profilometry systems are still an area of interest due to their advantages of fast operation, hardware simplicity, low cost, and simultaneous multi-measurement features. The proposed algorithm adaptively corrects the raw phase data for phase shifted system. As the simulation studies show the result that proposed method has an efficient execution time and low complexity, it is thought that it can be used in real-time applications.

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